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Sensitization and Extraordinary Persistence
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We propose a behavioral model in which an agent's attitude toward loss is affected by memories of prior losses. Due to the availability heuristic, memories of prior loss sensitize the agent and increase the weight assigned to prospective losses. Because memories of first-time experiences exhibit multi-decade persistence in recall, our model helps explain recent empirical findings that major events can have multi-decade effects on choices. We further demonstrate consistency with stochastic dominance, so that sensitized individuals will prefer distributions demonstrating first- and second-order stochastic dominance. In an overlapping generations version of Tirole's (2006) liquidity-scale framework, our model generates procyclical investment.

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"Sensitization, a form of learned fear in which a person or an experimental animal learns to respond strongly to an otherwise neutral stimulus." —Eric Kandel, 2000 Nobel Laureate in Physiology or Medicine

1. Introduction

In macroeconomics and finance, the preferences typically used assume that preferences regarding risk and loss are stable. Yet as noted by Dillenberger and Rozen (2015), recent empirical work suggests that an individual's attitude toward risk or loss may be affected by past experiences. For example, the literature has recently shown that shocks such as war, natural disasters, and even scary movies can cause individuals to act more risk-averse.[1]

Although there are existing models that generate time-varying risk or loss attitudes through time-varying habits and reference points, these habit- and reference point-based models have trouble explaining the magnitude of the duration in the changes in risk preferences. For example, both habits and reference points are thought to mostly adjust within a decade according to Campbell and Cochrane (1999) and Barberis, Huang, and Santos (2001), yet three recent works compellingly document that shocks to risk preferences may demonstrate multi-decade persistence. First, Malmendier, Tate, and Yan (2011) study CEOs from 1980 to 1994 and find that CEOs born before the Great Depression finance their corporate investments more conservatively than their post-depression contemporaries forty years later. Second, Schoar and Zuo (2011) find that CEOs who experienced a recession at age 24 (roughly when they entered the job market) made more conservative career choices throughout their careers. Third, Knüpfer, Rantapuska, and Sarvimäki (2014) document how individuals more exposed to the Finnish financial crisis in the 1990s were less likely to participate in the stock market over twelve years later.

We refer to the emerging finding of multi-decade persistence in shocks to risk preferences as *extraordinary persistence*, and this paper is motivated to use models of gain-loss preferences to understand potential behavioral causes of this

[1] See Kim and Lee (2014), Cameron and Shah (2015), and Guiso, Sapienza, and Zingales (2013).

extraordinary persistence. We believe that understanding this puzzling finding is important for macroeconomics and finance. Understanding the source of extraordinary persistence should be important to finance because of Cochrane's (2011, p. 1091) observation that most financial "puzzles and anomalies that we face amount to [variation in time- and/or risk-preferences] we do not understand." Due to the recent emphasis in incorporating financial markets within macroeconomic models, understanding the source of extraordinary persistence should also be important to macroeconomics.

In this paper, we develop a behavioral model of sensitization using gain-loss preferences to explain how time-varying loss attitudes demonstrating extraordinary persistence might arise. Our model of sensitization allows an individual to be sensitized by past realizations of loss: agents who experienced loss more frequently in their past will exhibit a greater tendency to focus on the potential for future loss. Consequently, these individuals should demonstrate a lower tolerance for bearing prospective losses in the future. Further, the model makes predictions regarding the persistence of changes in loss attitudes. According to our model, the duration to which a past loss will sensitize an individual will depend on the nature of the loss. In particular, our model predicts that losses characterized by first-time experiences during young adulthood are capable of persistently sensitizing an individual for over forty years, whereas the effects of most other types of loss should begin to fade within five years.

To model sensitization, we first consider a simple setting in which an agent who lives for two periods faces a lottery in the second period that is known to be independent of all first-period realizations. We start with the gain-loss utility developed by Kőszegi and Rabin (2006, 2007). In the absence of sensitization, the agent's choice in the second period depends on how the second period's distribution relates to the reference lottery, which in turn depends on that period's distribution. Because the lotteries are known to be independent, realizations from the first period do not affect the agent's second-period choice in the absence of sensitization.

In the presence of sensitization, preferences become path-dependent as past experiences of loss cause the agent to focus on the potential for future loss. Following Kőszegi and Rabin (2006, 2007), we assume that rational expectations inform the reference lottery each period. For tractability, we restrict the reference lottery to the first moments of the associated distributions. As a result, at the end of the first period the agent will compare all realizations to their expected values. All realizations short of the expected values trigger losses, whereas all realizations in excess of the expected values trigger gains. The agent's preferences in the second period will now depend on the frequency of loss in the first period. If losses are never experienced, a loss-averse agent's preferences are characterized by loss-aversion but not sensitization. If losses are experienced, a loss-averse agent's preferences are now characterized by loss-aversion and sensitization. Past losses are said to sensitize the agent, so that greater weight is placed on the potential for loss in the second period. Once sensitized, it is as if weight is transferred from the coefficient on gains to the coefficients on losses within the agent's gain-loss utility function.

We identify two distinct psychological channels that we believe are consistent with our model of sensitization described above. The first psychological phenomenon is referred to by Kahneman (2011) as the availability heuristic, and this channel generates time-varying attitudes toward loss. According to this heuristic, the weight that is assigned to a potential outcome (e.g., gain or loss) depends on the underlying availability of the outcome. Kahneman (2011) states that the availability of any outcome in turn depends on its emotional charge, vividness, ease of recall, and the degree to which the contemplated outcome is consistent with autobiographical memories. Based on Kahneman's description, we hypothesize that the more frequently that one experiences loss, the greater the ease of recall, vividness, and emotional charge these collective losses demonstrate. As frequent losses strengthen these availability factors, the availability heuristic will cause the individual to assign greater weight to future prospective losses.

The second psychological phenomenon is the reminiscence bump, and this channel generates the extraordinary persistence found in our model. According to psychologists studying the availability of remembered experiences, autobiographical memories of first-time experiences from young adulthood have been consistently shown to remain vivid, emotionally charged, and easily recalled forty years later.[2] In our model, these memories' persistent ease of recall, vividness, and emotional charge trigger a persistent distortion in availability, which in turn persistently affects the agent's attitude toward gain and loss through the availability heuristic.

Our model of sensitization is also influenced by Kandel's (2001) description, and this influence causes preferences to inherit three distinctive properties. First, preferences characterized by sensitization are *adaptive* because Kandel describes sensitization as a "form of learned fear." In our model, sensitization is said to be learned and adaptive because the degree of sensitization will depend on the agent's past experiences. Second, as a "form of learned fear," we interpret sensitization to be a fear of loss that is formed by prior experiences of loss. As it is illogical that an individual would develop a fear of gains, a second property of sensitization is *asymmetry*. In our model, first-time losses experienced when young will cause the individual to permanently shift her focus to the potential for future losses, whereas gains will not affect the agent's focus. Finally, following Kandel (2001) we model sensitization as a "form of learned fear" that operates

[2] See Rathbone, Moulin, and Conway (2008) for a recent summary of this literature and Rubin, Rahhal, and Poon (1998) for an earlier but more comprehensive review.

across stimuli. As an illustration of sensitization, Kandel argues that after hearing a noise similar to a gunshot, an individual will automatically become more alarmed by an otherwise neutral tap on the shoulder. Similarly, aversive realizations in our model will sensitize the agent to future losses even when the original aversive realizations are known to contain no informational content about the future.

Though sensitization includes these three unusual properties, the preferences of sensitization are shown to be well-behaved in a general setting to the extent that they exhibit consistency with the established notions of first- and second-order stochastic dominance. In order to demonstrate consistency, we start with Kőszegi and Rabin's (2006, 2007) gain-loss preferences. To enhance the tractability of the model of sensitization in general settings, we restrict the reference lottery to the first moment of the associated lottery's distribution.[3] We first show that these modified preferences will be consistent with the notions of first- and second-order stochastic dominance in the absence of sensitization. We then show that sensitization is also consistent with the notions of first- and second-order stochastic dominance, even under loss-neutrality.

After demonstrating that sensitization is well-behaved, this paper then studies sensitization's macroeconomic consequences using Tirole's (2006) liquidity-scale framework. In our version of Tirole's framework, financial entrepreneurs are assumed to manage investment projects while subject to potential cost overruns. If entrepreneurs' liquidity ratios sufficiently meet the realized cost overrun, their projects are successful and generate abundant capital per unit of investment. If

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[3] This restriction transforms Kőszegi and Rabin's (2006, 2007) preferences that depend on what Yitzhaki (1982) refers to as Gini's Mean Difference into preferences that depend on what Ogryczak and Ruszczyński (2001) refer to as Mean Absolute Semideviation. Because both general types of preferences have been demonstrated to be consistent with first-order and second-order stochastic dominance, our first-moment restriction will preserve consistency with stochastic dominance. In fact, our paper reproduces the required parameter restriction $\eta(\lambda - 1) < 1$ needed for consistency in the unrestricted preferences of Kőszegi and Rabin (2006, 2007) as demonstrated by Masatlioglu and Raymond (2014) as well as the Mean Absolute Semideviation preferences of Ogryczak and Ruszczyński (2001).

their liquidity ratios prove insufficient, their projects are unsuccessful and generate little capital. In this paper, the choice of liquidity ratio for each generation's representative entrepreneur characterizes how that generation manages risk at the intensive margin, whereas the scale of the investment project characterizes how that generation manages risk at the extensive margin.

In the presence of either sensitization or increases in loss-aversion, agents in the model are incentivized to reduce their sensations of net loss through distortions on the extensive (i.e., the scale of the investment project) and intensive margins (i.e., the project's liquidity ratio). Higher sensations of net loss reduce the agents' willingness to borrow, causing the scale of the project to fall on the extensive margin and is consistent with procyclical investment behavior.[4] With regard to the intensive margin, we find that the distortion introduced by sensitization critically depends on the probability of loss. When the probability of loss is low, increases in sensations of net loss cause agents to hold larger liquidity ratios. As a result, liquidity ratios become countercyclical when the probability of loss is low. Yet when the probability of loss is high, increases in sensations of net loss cause agents to hold smaller liquidity ratios. As a result, liquidity ratios become procyclical when the probability of loss is high. We believe that the counterintuitive property that increases in sensations of net loss may cause risk-taking to be procyclical at the intensive margin deserves further consideration in the macroeconomic literature.

Our model's theoretical predictions are generally consistent with the empirical literature. The model's prediction that borrowing falls after realized losses is consistent with the empirical finding by Malmendier, Tate, and Yan (2011) that CEOs born before the Great Depression are more hesitant to borrow to fund corporate investments than their post-depression contemporaries over forty years

[4] This sensation of net loss may include loss-aversion and sensitization as well as the interaction between the two.

later. To the extent that normal economic conditions are consistent with relatively low probabilities of loss, our model would predict that liquidity ratios are countercyclical, which is consistent with the empirical findings of Kato (2006).

The rest of this paper proceeds as follows. Section 2 presents a review of the literature. Section 3 presents a first-moment approximation of the preferences of Kőszegi and Rabin (2006, 2007). Section 3 then demonstrates that this approximation preserves consistency between loss-aversion and stochastic dominance. Section 4 develops the preferences of sensitization and, as this paper's first main contribution, discusses how these sensitized preferences induce time-varying risk preferences through time-varying attitudes toward loss. Section 4 then develops this paper's second contribution by demonstrating that sensitization is consistent with stochastic dominance under slightly more restrictive parameter assumptions. As this paper's final contribution, Section 5 introduces an overlapping generations version of Tirole's (2006) liquidity-scale framework to study sensitization's effects on the cyclicality of investment and liquidity using comparative statics following an increase in realized losses. Section 6 concludes with general findings, while the appendix contains the proofs for the propositions of consistency of first-order stochastic dominance with both loss-aversion and sensitization.

2. Relationship to the Economic Literature

This paper is motivated by the finding that major economic shocks can affect risk preferences at the microeconomic level for decades. Four papers effectively document the persistence of these shocks. First, Malmendier, Tate, and Yan (2011) study CEOs from 1980 to 1994 and find that CEOs born before the Great Depression finance their corporate investments more conservatively than their post-depression contemporaries. Second, Knüpfer, Rantapuska, and Sarvimäki (2014) document how individuals more exposed to the Finnish financial crisis in the 1990s were less likely to participate in the stock market twelve years later.

Third, Schoar and Zuo (2011) find that CEOs who experienced a recession at age 24 (roughly when they entered the job market) made more conservative career choices throughout their careers. Finally, using a Markov-Switching model to estimate shifts in aggregate risk regimes, Gordon and St-Amour (2000) find that risk preferences vary over time at the *aggregate* level and that shifts in aggregate risk regimes persist for roughly ten years.

Traditionally, time-varying risk preferences have been modeled through time-varying discount rates. Given the recognized importance of time-varying discount rates to understanding financial anomalies, the discipline has expended considerable effort toward understanding the source of time-varying discount rates.[5] Perhaps the most common approach, and the approach employed in this paper, is to use time-varying risk preferences to generate time-varying discount rates. Yet many of the seminal works in the time-varying risk preferences literature, such as Campbell and Cochrane's habit-based model (1999) and Barberis, Huang, and Santos' (2001) reference-based model, largely preceded the recent empirical findings discussed in the four papers highlighted above. It is therefore not surprising that the degree of persistence generated in these works would appear insufficient to explain the multi-decade persistence that has only recently been documented.[6]

Within the literature on time-varying risk and loss attitudes, the paper most similar to ours is Dillenberger and Rozen's (2015), yet there are also important differences. This paper is similar to ours to the extent that both generate time-varying risk preferences consistent with the psychological phenomena

[5] One of the earliest and most important financial anomalies was the finding of excess volatility by Shiller (1981) and LeRoy and Porter (1981). See Farmer, Nourry, and Venditti (2012) and Dillenberger and Rozen (2015) for a comprehensive review of this literature.

[6] Since habits and reference points are generally believed to adjust within a decade, it is not clear that any model generating time-varying risk aversion through either sticky habits or sticky reference points alone would be capable of generating multi-decade persistence of shocks to risk preferences.

Dillenberger and Rozen refer to as the reinforcement and primacy effects.[7] The differences between the two papers stem from differences in motivation. Whereas Dillenberger and Rozen primarily focus on developing a model consistent with the reinforcement and primacy effects, this paper primarily focuses on understanding the cause of the extraordinary persistence found in the empirical literature.

The use of loss-aversion as the foundation for our paper allows us to identify the distinct behavioral and psychological phenomena that generate extraordinary persistence. Identifying these phenomena allow for predictions regarding the reinforcement and primacy effects that are more nuanced than found in Dillenberger and Rozen (2015). First, our model predicts that the reinforcement effect is moderated by the availability heuristic. Our model therefore predicts that past economic losses such as the Great Depression should affect economic choices more than other types of losses such as wars. This is because memories of economic losses should be more available and salient than memories of other losses when contemplating economic choices. This prediction may explain why the effects of the Great Depression on economic choices as documented by Malmendier, Tate and Yan (2011) appear more consistently statistically significant than the effects of the Korean War on economic choices as documented Kim and Lee (2014). Second, our model predicts that the primacy effect demonstrates temporal discontinuities. Whereas Dillenberger and Rozen's model allows all shocks to permanently affect risk preferences, our model suggests that the effect on risk preferences depend on the availability of the memory of the shock. Because only memories of first-time experiences or experiences from young adulthood have been shown to remain persistently

[7] In our model, the reinforcement effect would suggest that losses make one more loss-averse, whereas the primacy effect suggests that first-time or early losses have larger and more persistent effects on risk and loss attitudes.

available with other memories generally fading in five years time, our model predicts that only first-time experiences or experiences from young adulthood are likely to permanently affect risk and loss attitudes.

Although this paper's primary contribution is directed toward the time-varying risk preferences literature, it also relates to other research in economics, such as the availability heuristic, the Two Systems model, and Shiller's (2014) Nobel Lecture discussion of the sociological forces that he believes drive animal spirits and bubbles.

Starting with the economic literature on the availability heuristic, we note that although this literature is exceedingly small, there are two notable works related to ours. First, Akerlof and Yellen (1987) similarly discuss the potential macroeconomic implications of the availability heuristic and other behavioral phenomena. Second, Fuster, Laibson, and Mendel (2010) formally model how similar psychological phenomena, including the availability heuristic, could bias expectations and generate excess volatility in asset markets. Although this paper deviates from these two by identifying the availability heuristic with loss attitudes instead of expectations, we follow the bounded rationality approach of these two works by developing an equilibrium in which rational expectations and availability biases simultaneously coexist. Specifically, by identifying the availability heuristic as potentially contributing to long-run sensitization (a form of learned fear), we argue that people may be aware that their risk preferences are biased toward magnifying losses and yet be unable to correct these biases. For example, individuals scarred by an event like the Great Depression during their youth may know that their fear of future loss is irrational. Yet, these irrational fears may still bias individual behavior as distorted fixations on loss may not be easily addressed by Bayes' rule.

This paper's use of the availability heuristic relies on a psychological model of human decision-making endorsed by Kahneman (2011). This reliance causes our work to be related to the literature studying what Kahneman refers to as the Two Systems model. Kahneman argues that people's decisions are informed by both logic-driven beliefs (e.g., rational expectations) and emotions (e.g., distorted fears of loss). In his view, emotions are influenced by the availability heuristic, which in this paper is modeled through a distorted focus on loss. We are not the first to consider the economic implications of what Kahneman (2011) refers to as the Two Systems model, but to the best of our knowledge we are the first to explicitly analyze the implications of the Two Systems model on risk and loss attitudes. Other authors have explicitly used a Two Systems framework to study primarily microeconomic phenomena. For example, Brocas and Carrillo (2008) use a Two Systems model to study impatience, and Fudenberg and Levine (2006) use it to study self-control.

Finally, this paper studies how a generation's collective experiences may shape future economic decision-making, which parallels the discussion of animal spirits in Shiller's (2014) Nobel Lecture. While discussing sociological factors contributing to animal spirits' ability to affect asset prices, Shiller cites Durkheim's notion of "the 'collective consciousness,' that represents the shared beliefs, attitudes, and moral judgments that characterize a time" (p. 1496). He further cites Halbwachs' notion of "the 'collective memory,' the set of facts that are widely remembered at any point of time, but that are forgotten eventually if word of mouth and active media do not perpetuate their memory" (pp. 1496-1497). This paper contributes to Shiller's discussion by suggesting a generation's persistent memories of first-time economic losses from young adulthood (i.e., part of their "collective memory") may generate distortions in loss attitudes, (i.e., alterations in their "collective consciousness") and thereby affect asset prices.

3. Sufficient Conditions for Consistency Between Loss-Aversion and Stochastic Dominance

In this section we first demonstrate consistency between loss-aversion and first- and second-order stochastic dominance when reference lotteries are approximated by their first moments.[8] That is, loss-averse individuals will prefer distributions that first- and second-order stochastically dominate others. Then in the next section, we demonstrate consistency between sensitization and first- and second-order stochastic dominance when reference lotteries are approximated by their first moments.

For tractability, we approximate the reference lottery employed by Kőszegi and Rabin (2006, 2007) with its first-order moment. We employ this first-moment approximation because it enhances the tractability of our model of sensitization in general settings while still allowing reference lotteries to be determined by expectations as argued by Kőszegi and Rabin (2006).[9,10] To the extent that actions are largely driven by heuristics as argued by Kahneman (2011), this first-moment approximation may even better characterize the bounded-rationality of real-world agents.

Starting with the necessary notation, suppose an individual (she) is loss-averse and considers two lotteries over some monetary payoffs y with cumulative

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[8] Although Kőszegi and Rabin (2007) and Masatlioglu and Raymond (2014) have already provided proofs of consistency with stochastic dominance, this section establishes that consistency is still preserved under the first-moment approximation that will be used in the model of sensitization.

[9] This tractability assumption is not without costs. Besides being less general for non-binary distributions, one additional consequence of this assumption is that it will make individuals more tolerant of risk. Consider an example of a job-market candidate contemplating three potential salary offers, $y_H > y_M > y_L$ and in which the middle observation coincides with the mean of the distribution, $y_M = \mu$. If the reference lottery is determined by its first moment, Equation $(3.2a)$ would predict that the individual would not experience any gain or loss when y_M is offered because the individual's expectation μ was exactly met. In contrast, if the entire reference lottery is considered, then according to Equation (3.1a), the individual would compare y_M with both y_L and y_H and experience a net loss.

[10] Our preferences of sensitization do not require this tractability assumption. Because our model of sensitization can be thought to shift weight from gains to losses in proportion to the availability of losses, we could also construct a (less tractable) model of sensitization using the unrestricted reference lotteries of Kőszegi and Rabin (2006, 2007). We prefer this first-moment approximation strictly for its enhanced tractability in general settings.

density functions $F(y)$ and $G(y)$ and probability density functions $f(y)$ and $g(y)$, respectively. Using the reference lottery concept of Kőszegi and Rabin (2006, 2007) and assuming loss-aversion, risk-neutrality, and constant sensitivity, when $\hat{y}$ is realized the individual's sense of gain depends on how $\hat{y}$ compares to all realizations (probability-weighted) that are relatively more adverse than $\hat{y}$.[11] If the lottery with cdf $F(y)$ is chosen, then the individual achieves gain: $\eta \int_{-\infty}^{\hat{y}} (\hat{y} - y) f(y) dy,$ in which η can be interpreted as the gain-loss coefficient. In contrast, the sense of loss triggered by realization $\hat{y}$ depends on how $\hat{y}$ compares to all realizations (probability-weighted) that are more favorable than $\hat{y}$. If the lottery with cdf $F(y)$ is chosen, then the individual achieves loss: $\eta\lambda \int_{\hat{y}}^{\infty} (\hat{y} - y) f(y) d(y),$ in which $\lambda > 1$ captures the coefficient of loss-aversion. Thus, when $\hat{y}$ is realized, the sense of net loss experienced is given by: $\eta \int_{-\infty}^{\hat{y}} (\hat{y} - y) f(y) dy + \eta\lambda \int_{\hat{y}}^{\infty} (\hat{y} - y) f(y) d(y)$. The total *ex-post* utility, which combines the normal risk-neutral consumptive utility y with the gain-loss utility is therefore:

$$(3.1a)\ U(\hat{y})|F = \hat{y} + \eta \int_{-\infty}^{\hat{y}} (\hat{y} - y) f(y) dy + \eta\lambda \int_{\hat{y}}^{\infty} (\hat{y} - y) f(y) dy.$$

As revealed in Equation $(3.1a)$, Kőszegi and Rabin's (2006, 2007) model requires the entire distribution to determine the reference lottery from which gains and losses are calculated.

Whereas Equation $(3.1a)$ demonstrated how the reference lottery determined the *ex-post* utility, we wish to show that calculations of expected utility are somewhat less tractable and depend on a double-integral. For each possible realization $\hat{y}$, the individual must weight $\hat{y}$'s relative contribution to expected

[11] Constant sensitivity corresponds to Assumption A3' in Kőszegi and Rabin (2006).

utility by its probability. If the lottery with cdf $F(\cdot)$ is chosen, then the individual's expected utility is given by:

$$(3.1b)\ E[U(\hat{y})|F]$$
$$= E(\hat{y})$$
$$+ \int_{-\infty}^{\infty} f(\hat{y})$$
$$* \left(\eta \int_{-\infty}^{\hat{y}} (\hat{y} - y) f(y) dy + \eta\lambda \int_{\hat{y}}^{\infty} (\hat{y} - y) f(y) d(y) \right) d\hat{y}.$$

To enhance tractability, we replace Kőszegi and Rabin's (2006) reference lotteries in equations $(3.1a)$ and $(3.1b)$ with their first-moment approximations. We then demonstrate in Propositions 0a and 0b that the gain-loss preferences involving this first-moment approximation are still consistent with first- and second-order stochastic dominance. For the rest of the analysis, when the individual chooses the lottery with cdf $F(y)$, the first moment $\mu_F = \int_{-\infty}^{\infty} y * f(y) dy$ will be substituted for the agent's reference lottery. When $\hat{y}$ is realized, the agent's total *ex-post* utility is now approximated by:

$$(3.2a)\ U(\hat{y})|F = \begin{cases} \hat{y} + \eta(\hat{y} - \mu_F)\ if\ \hat{y} > \mu_F \\ \hat{y} + \eta\lambda(\hat{y} - \mu_F)\ if\ \hat{y} \le \mu_F \end{cases}.$$

Following Kőszegi and Rabin's (2006, 2007) concepts of *preferred personal equilibrium (PPE)* and *choice-acclimating personal equilibrium (CPE)*, if the agent chooses lottery $F(\cdot)$, her expected utility is given by:

$$(3.2b)\ E[U(\hat{y})|F] = \mu_F + \eta\lambda \int_{-\infty}^{\mu_F} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y} + \eta \int_{\mu_F}^{\infty} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y}.$$

The differences between the pair of equations $(3.2a)$ and $(3.2b)$ and the pair of equations $(3.1a)$ and $(3.1b)$ are entirely attributable to differences in the reference concepts used. Equations (3.1a) and (3.1b) utilize the reference lottery concept of Kőszegi and Rabin (2006, 2007), whereas Equations $(3.2a)$ and

$(3.2b)$ utilize the reference lottery's first-moment approximation to enhance tractability.

Having now established expressions for the agent's utility for loss-aversion when the agent's reference lottery is approximated by its first moment, we will demonstrate that if F first- or second-order stochastically dominates G, then the individual's expected utility under F is greater than under G. These proofs are important for the paper as our model of sensitization substitutes a first-moment approximation for the reference lottery. The following proofs will demonstrate that this approximation is still well-behaved to the extent that consistency with stochastic dominance is preserved. We start by stating the result for first-order stochastic dominance in Proposition 0a (see the appendix for the proof) and then prove the result for second-order stochastic dominance under Proposition 0b.

Proposition 0a. If F and G are two lotteries with supports bounded below and if F demonstrates strict first-order stochastic dominance over G, then F will be strictly preferred to G by any agent with preferences characterized Equation (3.2b) under moderate levels of loss-aversion.[12]

Proposition 0a states that as long as the sensation of net loss is not too severe, then loss-averse agents should generally prefer distributions that first-order stochastically dominate others. Proposition 0a implicitly assumes Kőszegi and Rabin's (2007) concept of *choice-acclimating personal equilibrium* instead of their (2006) concept of *preferred personal equilibrium.*[13] Because agents in a *choice-acclimating personal equilibrium* have greater incentive to deviate to a

[12] We require the parameter restriction $\eta(\lambda - 1) < 1$, which is consistent with both the reference lottery findings of Masatlioglu and Raymond (2014) as well as the Mean Absolute Semideviation findings of Ogryczak and Ruszczyński (2001).

[13] In a *choice-acclimating personal equilibrium*, the reference lottery for an off-the-equilibrium-path choice varies with the choice considered, whereas in a *preferred personal equilibrium* the reference lottery is fixed by the on-the-equilibrium-path choice.

first-order stochastically dominated distribution, the sufficient conditions identified operate across both types of equilibria.[14]

Proposition 0b states a similar result for second-order stochastic dominance but does not require the parameter restriction $\eta(\lambda - 1) < 1$:

Proposition 0b. If F and G are two lotteries with identical means, if F and G both have bounded supports, and if F second-order stochastically dominates G, then F will be (weakly) preferred to G by any agent with preferences characterized by Equation (3.2b).

Proof: The proof follows directly from property 6.D.2 from Mas-Colell, Whinston, and Green (1995) that $\int_{-\infty}^{\mu} F(y) - G(y)dy \leq 0$ for $\mu_G = \mu_F = \mu$ and from Equations $(A1)$ and $(A2)$ in the appendix of this paper.[15]

Compared to the findings for first-order stochastic dominance, the findings for second-order stochastic dominance are stronger in that second-order stochastic dominance does not require any parameter restrictions. Yet, the findings for second-order stochastic dominance are weaker because they do not guarantee strict preference.[16]

4. Sufficient Conditions for Consistency Between Sensitization and Stochastic Dominance

The preceding section demonstrated consistency between loss-aversion and first- and second-order stochastic dominance when reference lotteries are approximated by their first moments. This section will first develop a model of

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[14] Unlike a *preferred personal equilibrium,* in a *choice-acclimating personal equilibrium*, the agent's deviation to a dominated distribution lowers the reference point. This lower reference point increases the agent's utility and gives the agent greater incentive to deviate to a first-order stochastically dominated distribution.

[15] Note that the derivation of these equations does not require first-order stochastic dominance. Instead, these equations are true for any distribution.

[16] To show that strict preference does not hold, note that it is possible to construct two lotteries in which F demonstrates strict second-order stochastic dominance over G and have a loss-averse agent indifferent between the two. This unexpected result stems from the fact that the agent only cares about variation below the mean in Equation (A1). Should G represent an *elementary increase in risk* from F but with $F(\hat{y}) = G(\hat{y})$ for all $\hat{y} < \mu_F = \mu_G$, then the agent would be indifferent between the two lotteries even though F demonstrates strict second-order stochastic dominance over G.

sensitization and then show that sensitization is also consistent with first- and second-order stochastic dominance.

4.1 Preferences Capturing Sensitization

This subsection introduces modifications of the preferences in Equation $(3.2b)$ to capture sensitization. At a broad level, we model sensitization as a learned fear of loss. In the model, agents who are exposed to frequent losses when young become overly fixated on the possibility of loss. The subsection concludes by discussing the implications and justifications for the introduced modeling choices.

To capture the notion that experiences of loss sensitize individuals to fear future losses and to capture Kandel's (2001) description of sensitization as "a form of learned fear" that can operate across stimuli, this paper's model of sensitization operates through the *frequency* of loss experienced during early adulthood.[17] Specifically, let x represent a comprehensive summary variable representing all variables relevant for the utility function. Let $H(x_t)$ denote the *ex-ante* cumulative density function with mean μ_H of this comprehensive summary variable during period t. Then $\widehat{H}_t(\mu_H)$ captures the average frequency of loss that is realized across all variables in the utility function for an individual young during period t. Given $\widehat{H}_t(\mu_H)$, we say that the individual's expected utility over some potentially new variable y_{t+1} when older during period $t+1$ will be biased toward future loss whenever $\widehat{H}_t(\mu_H) > 0$. If $\widehat{H}_t(\mu_H) = 1$, we say that the individual will be maximally sensitized. To capture these preferences more specifically, assume that if $\widehat{H}_t(\mu_H) = 0$, then the individual remains non-

[17] We model sensitization through the frequency and not magnitude of past losses because we model sensitization using the availability heuristic, which Kahneman (2011) argues operates through biased uses of probabilities. To the extent that a major traumatic episode triggers frequent sensations of loss, the frequency-based approach may also be consistent with a magnitude-based approach.

sensitized and the expected utility associated with lottery F is given by Equation $(3.2b)$.[18]

$$E[U(y_{t+1})|F] = \mu_F + \eta\lambda \int_{-\infty}^{\mu_F} (y_{t+1} - \mu_F) f(y) dy + \eta \int_{\mu_F}^{\infty} (y_{t+1} - \mu_F) f(y) dy.$$

We now introduce the availability parameter $\alpha \in [0,1]$ to capture the relative degree to which an individual may be sensitized by losses on x. Since sensitization is generated in our model by the availability heuristic, the degree of sensitization will depend on the availability of the experienced loss when contemplating future decisions.[19] $\alpha = 0$ corresponds to an individual who is either immune to sensitization or whose losses experienced on x during period t are not available when considering potentials gains and losses on y during period $t + 1$. $\alpha = 1$ corresponds to an individual who is maximally susceptible to sensitization and whose losses on x experienced during period t were maximally available when considering potential gains and losses on y during period $t + 1$. When $\widehat{H}_t(\mu_H) = 1$, the individual will be maximally sensitized given her α and will have preferences given by:

$$E[U(y_{t+1})|F] = \mu_F + \eta(1-\alpha) \int_{\mu_F}^{\infty} (y_{t+1} - \mu_F) f(y) dy$$
$$+ \eta\lambda(1+\alpha) \int_{-\infty}^{\mu_F} (y_{t+1} - \mu_F) f(y) dy.$$

For more intermediate levels of $\widehat{H}_t(\mu_H)$, the degree of sensitization is proportional to the frequency of loss when young:

18 With lottery F having cumulative density function $F(y_{t+1})$ and mean μ_F.

19 Kahneman (2011) argues that emotional charge, vividness, and ease of recall are primary determinants of availability.

$$(4.1)\ E[U(y_{t+1})|F] = \mu_F + \eta\left(1 - \alpha\widehat{H}_t(\mu_H)\right)\int_{\mu_F}^{\infty}(y_{t+1} - \mu_F)f(y)dy + \eta\lambda\left(1 + \alpha\widehat{H}_t(\mu_H)\right)\int_{-\infty}^{\mu_F}(y_{t+1} - \mu_F)f(y)dy.$$

Equation (4.1) comprehensively captures the preferences of sensitization for a given lottery F, and the chosen specification of sensitization demonstrates six implicit properties worth discussing. We first explicitly introduce the four new properties of focus-shifting, Bayesian non-updatability, boundedness, and monotonicity before expanding upon the two previously-discussed properties of being availability-driven and extraordinarily persistent.[20]

First, sensitization is said to be focus-shifting. In this model of sensitization, past experiences of loss cause an individual to focus her attention away from expected gains to expected losses. Although one could model past experiences of loss as lowering the reference points, this would serve to increase the agent's utility and therefore does not seem realistic. One could also assume that sensitization primarily operates by distorting the first term in Equation (4.1), "the consumption utility component" or the component of the agent's expected utility that is independent of gains and losses. Yet, it would no longer be clear how sensitization would be able to operate *across* stimuli in the absence of informational content if sensitization operated by distorting the agent's consumption utility.

There are two further advantages of modeling sensitization's availability biases as a shift in focus instead of a bias in probability as discussed by Kahneman (2011). One advantage is that it leads to our second discussion-worthy property

[20] A seventh but less important property stems from the choice to model sensitization as an equal shift in focus from gains to losses. As a result, Equation (4.1) utilizes the same α across both gain and loss terms in the utility function, but this is not necessary as the parameter restrictions used in the next subsection's proofs could easily be altered to address asymmetry in α.

of Bayesian non-updatability, and this property allows for sensitization and rational expectations to easily coexist within a boundedly-rational model. In the model, agents' rational expectations inform both consumption utility and the reference points, but these expectations prove insufficient to update the irrational fixation or focus on loss. Therefore, sensitization's shift in focus is not able to be updated in a Bayesian manner. In contrast, if sensitization were interpreted to operate through biased likelihoods, then agents should be able to replace their biased likelihoods with their rationally-calculated ones, precluding sensitization from affecting preferences. Another advantage is that this paper's approach as a shift in focus requires fewer ad hoc assumptions to demonstrate consistency with first- and second-order stochastic dominance.

As a third discussion-worthy property, the degree of sensitization is said to be bounded. When $\alpha < 1$, the individual will never completely ignore the potential for gains. Only if $\alpha \widehat{H}_t(\mu_H) = 1$ would the individual completely shift her focus from gains to losses, and the restriction that $\alpha \in [0,1]$ precludes the individual from shifting the focus even further. Furthermore, the non-negativity of α implies that focus can be shifted to losses but not to gains, which is consistent with an asymmetrical model of sensitization based on a learned fear of loss.

Fourth, these preferences are monotonically increasing. Individuals who experienced losses more frequently are predicted to exhibit a greater distorted focus toward loss. This monotonicity property may explain why persistent shocks generating frequent experiences of loss (e.g., the Great Depression) have larger long-run impacts on future risk preferences than one-time shocks generating negligible losses (e.g., seeing a favorite character suffer an adverse shock in a scary movie).

Fifth, sensitization is said to be availability-driven because sensitization operates through the availability parameter α, and the presence of this parameter differentiates sensitization from adaptive loss-aversion. α governs the degree to which experiences of losses in one dimension of utility sensitize decision-making in other dimensions. Losses that are more available are consistent with a larger α, whereas less available losses are consistent with a smaller α. Since Kahneman (2011) argues that salience affects availability, this specification predicts that non-economic shocks are less likely to sensitize future economic decision-making. This prediction helps differentiate our model from a more general model of adaptive loss-aversion, in which non-economic shocks would be equally likely to sensitize future economic decision-making.

Sixth, and as previously emphasized, the shifts in risk preferences resulting from first-time shocks during young adulthood are predicted to be extraordinarily persistent. This persistence is due to the presence of the availability parameter α. Because memories of first-time experiences have been shown to be easily recalled, vivid, and emotionally charged decades later, the persistence of these availability factors for memories of first-time experiences during young adulthood predicts that memories of these experiences will generate decades-long distortions in risk and loss attitudes.

4.2 Sensitization and Stochastic Dominance

This subsection will demonstrate that the sensitization preferences specified by Equation (4.1) will be consistent with first- and second-order stochastic dominance under the stricter sufficient condition that $\eta\lambda < 1/2$.[21] As before, let $H(x_t)$ denote the *ex-ante* cumulative density function of x at time t and suppose that the agent can choose between lotteries F and G over some variable y at time $t+1$. Suppose further that these lotteries F and G have cumulative density functions given by $F(y_{t+1})$ and $G(y_{t+1})$, respectively. Finally, suppose that the

[21] Instead of the looser sufficient condition $\eta(\lambda - 1) < 1$ required for consistency between loss-aversion and stochastic dominance in the previous subsection. If losses have twice the magnitude of gains so that $\lambda = 2$, this stricter restriction requires $\eta < 1/4$. That is, the weight of the gain-loss utility component must be less than a quarter of the weight of the standard consumption utility component.

probability density functions of these three lotteries are given by $h(x_t), f(y_{t+1})$, and $g(y_{t+1})$, in which the probability density function corresponds to the cumulative density function of the same letter. Given experienced frequency of loss $\widehat{H}(\mu_H)$ at time t during the agent's young adulthood, Equation (4.1) denotes the agent's expected utility under lottery F, and Equation (4.2) below will represent the agent's expected utility under lottery G:

$$(4.2)\ E[U(y_{t+1})|G]$$

$$= \mu_G + \eta\left(1 - \alpha\widehat{H}_t(\mu_H)\right)\int_{\mu_G}^{\infty}(y_{t+1} - \mu_G)g(y)dy$$

$$+ \eta\lambda\left(1 + \alpha\widehat{H}_t(\mu_H)\right)\int_{-\infty}^{\mu_G}(y_{t+1} - \mu_G)g(y)dy.$$

Given Equations (4.1) and (4.2), we wish to show that $E\big(U(y_{t+1}|F)\big) \geq E\big(U(y_{t+1}|G)\big)$ whenever F first- or second-order stochastically dominates G. Proposition 1a is stated immediately below and the proof is provided in the appendix.

Proposition 1a. If F and G are two lotteries with supports bounded below and if F demonstrates strict first-order stochastic dominance over G, then F will be strictly preferred to G by any agent sensitized according to Equation (4.1) under moderate levels of loss-aversion.[22]

The sufficient condition identified in Proposition 1a relating to sensitization requires stricter restrictions for the parameters η, λ than used in the proof of Proposition 0a without sensitization. However, unlike Proposition 0a, Proposition 1a does not require loss-aversion $(\lambda > 1)$ to guarantee strict preference of the strictly dominant lottery. Even loss-neutral but sensitized agents will strictly prefer the strictly dominant lottery. In addition, the proof assumes extreme levels

[22] As mentioned, it is sufficient for the proof that both η and λ are bounded such that $\eta\lambda < 1/2$. This restriction is slightly stricter than the earlier sufficient condition $\eta(\lambda - 1) < 1$ used for Proposition 0a.

of sensitization, $\alpha = 1$. Under less extreme levels of sensitization than $\alpha = 1$, one could derive weaker restrictions.

In contrast to the results of first-order stochastic dominance, the sufficient conditions for second-order stochastic dominance do not require additional restrictions on the parameters η, λ. Yet as before, the results for second-order stochastic dominance in Proposition 1b do not guarantee that an agent will strictly prefer the strictly dominant distribution. Following Mas-Colell, Whinston, and Green (1995), we assume that the two distributions have identical means. Proposition 1b is listed below and its proof stems directly from the property of second-order stochastic dominance that $\int_{-\infty}^{\mu} F(y) - G(y) dy \leq 0$ for $\mu = \mu_F = \mu_G$ and from Equation $(A6)$ in the Appendix.[23]

Proposition 1b. If F and G are two lotteries having bounded supports and identical means and if F second-order stochastically dominates G, then F will be (weakly) preferred to G by any agent sensitized according to Equation (4.1).

5. Financial Model with and without Sensitization

Having established properties of sensitization in a general setting, this section proceeds to study sensitization's macroeconomic consequences. This section first develops an overlapping-generations version of Tirole's (2006) liquidity-scale framework when agents are loss-neutral and non-sensitized. This section then develops a model with sensitization to study the comparative statics of sensitization from a positive but not normative perspective.[24]

5.1 Non-sensitized Financial Model under Loss-Neutrality

This section studies how sensitization affects risk choices and key macroeconomic variables, such as liquidity ratios and investment, through

[23] Note that Equation $(A6)$ is true for any two distributions and does not require first-order stochastic dominance to derive.

[24] The analysis of investment and liquidity distortions is positive but not normative. The analysis does not explicitly identify inefficiencies for policymakers to address because agents in our model are still maximizing loss-adjusted returns.

generational shifts in risk preferences. Since we could not find a suitable model in the existing literature to serve as a comprehensive benchmark, the benchmark model combines elements from the models of Bernanke and Gertler (1989), Tirole (2006), and Romer (1986). We refer to this benchmark as the non-sensitized benchmark because agents are assumed to be loss-neutral and immune to sensitization with $\lambda = 1$ and $\alpha = 0$.

Like all of the models considered in this section, this benchmark is a partial equilibrium analysis of a financial sector populated by financial entrepreneurs. Although an external household sector and external production sector are both present, the details determining equilibria in these two sectors are exogenous to the model as presented in this paper. In this paper, households simply provide funds to financial agents with a given upwardly-sloped savings function $S(R)$. For tractability, we assume a Romer (1986) production function in which the return on capital is fixed, and we further assume perfect capital depreciation within each period. Consequently, firms buy units of capital from financial entrepreneurs with time-invariant price q equal to a time-invariant marginal product of capital.

A timeline of events in the benchmark model is given below. The events identified in the timeline will be discussed in-depth later in this section.

1. A unit mass of entrepreneurs of generation t arrives.

2. The investment projects of generation $t-1$ are realized and aggregate capital K_t is formed.

3. Each unit of capital K_t is bought at price q. Entrepreneurs of generation $t-1$ consume c_t^e before expiring.

4. Production occurs with households and entrepreneurs inelastically providing one unit of labor for wage w_t.

5. Households offer upward-sloping savings schedule $S(R_t)$ to financial entrepreneurs.

6. Of the total funds i_t available for investment, the entrepreneur allocates fraction l_t toward liquid assets and invests the remaining funds $(1 - l_t) * i_t$ in the investment project.

7. An idiosyncratic cost overrun ρ_t is realized. The cost overrun is paid if $l_t \geq \rho_t$ and the project is said to be successful. Following success, the entrepreneur will have capital next period $k_{t+1} = \kappa_H(1 - l_t)i_t$. When $l_t < \rho_t$, the cost overrun cannot be paid and the project is said to fail. Following failure, the entrepreneur will have capital $k_{t+1} = \kappa_L(1 - l_t)i_t$ next period. Given the unit mass of entrepreneurs, $K_{t+1} = k_{t+1}$.

As indicated in the timeline, the role of financial entrepreneurs is to convert savings into physical capital. Like Bernanke and Gertler (1989), these financial entrepreneurs manage their investment projects.[25] Entrepreneurs borrow amount b_t from households and pay rate of interest R_t. This borrowing is assumed to take the form of bonds with R_t independent of the investment project's realization.

To generate capital for the economy, entrepreneurs devote their entire wage income (w_t) to savings and combine their savings with money borrowed from households (b_t) to generate k_{t+1} units of capital. Because they are assumed to not consume during their period of entry, their consumption next period c_{t+1}^e is given by:

$$c_{t+1}^e = qk_{t+1} - R_t b_t.$$

---

[25] Unlike Bernanke and Gertler, this paper uses a partial equilibrium analysis for enhanced tractability.

To study how sensitization affects risk choices, investment projects will have the characteristics described by Tirole (2006, p. 209-210). Central to his model is a tradeoff between liquidity and scale. For simplification, he assumes that investment projects are identical across each generation's representative entrepreneur and that they exhibit constant returns to scale. In both Tirole's model and ours, risk stems from the presence of liquidity shocks. These shocks take the form of a potential cost overrun, so that entrepreneurs' initial investment outlay may require a further cash injection ρ_t to successfully bring the project to completion.[26]

In order to manage the risk of a cost overrun, entrepreneurs will need to arrange for access to liquid assets. If we think of the lending w_t by the entrepreneur to the project as junior debt and the external borrowing b_t as senior debt, then we can think of the choice of liquidity as being captured a liquidity ratio $l_t \in [0,1]$, defined as the amount of liquid assets as a percentage of total liabilities. Because $i_t = w_t + b_t$ measures both total liabilities and the size of the investment project, the liquidity ratio also represents the fraction of the project's funding that is allocated toward liquid assets. The remaining fraction, $(1 - l_t)$ will be allocated toward the (illiquid) initial investment outlay, $(1 - l_t) * (w_t + b_t)$.

When a cost overrun occurs, there are two possible outcomes depending on the liquidity ratio previously secured by the entrepreneur. If that generation's representative entrepreneur (she) has sufficient liquid assets and pays the cost overrun, then the project is said to be successful and $k_{t+1} = \kappa_H * (1 - l_t) * i_t$ units of capital are generated next period. If she does not pay the cost overrun, then the project is said to be unsuccessful and only $k_{t+1} = \kappa_L * (1 - l_t) * i_t$ units of capital are generated, with both $\Delta\kappa = \kappa_H - \kappa_L$ and κ_L positive. Like Tirole

[26] These cost overruns may be due to a variety of sources: design flaws, strategic behavior by a contracted party seeking to renegotiate the contracted terms, or from the inability to secure a key input.

(2006), the supply of liquid assets is perfectly elastic with the return to liquid assets normalized to zero.[27] Additionally, these assumptions imply that entrepreneurs will pay the cost overrun when able and allow us to focus on the entrepreneur's *ex-ante* liquidity choice, this section's main concern.

Similarly to Tirole's model, the properties of cost overruns are as follows. First, the size of the cost overrun is proportional to the amount of funding acquired by the entrepreneur, i_t. Second, the magnitude of the shock $\rho_t \in [0,1]$ has probability density function $f(\rho_t)$ and cumulative density function $F(\rho_t)$. Third, liquidity arrangements must be made prior to the shock's realization. Finally, entrepreneurs must pay all cost overruns from their liquid assets.[28]

Together, the amount of liquidity available and the size of the cost overrun (or liquidity shock) determine the project's outcome. If the liquidity ratio is at least as large as the realized shock, $l_t \geq \rho_t$, the project is successful. This outcome occurs with probability $F(l_t)$. Otherwise when $l_t < \rho_t$, the project fails. Failure occurs with probability $\left(1 - F(l_t)\right)$.

Risk-neutral entrepreneurs use liquidity to maximize their expected net returns. These returns depend on the price of capital, the expected quantity of capital produced, and borrowing costs:

$$(5.1) \quad \underbrace{q}_{\text{price of capital}} \underbrace{\overbrace{(1 - l_t)\left[\kappa_H \mathrm{F}(l_t) + \kappa_L\left(1 - \mathrm{F}(l_t)\right)\right]}^{\mathrm{E}[\kappa(l_t)]} * \overbrace{(w_t + b_t)}^{i_t}}_{E[k_{t+1}] = \text{expected units of capital generated}}$$
$$- \underbrace{R_t b_t}_{\text{borrowing costs}}.$$

[27] Note that in the absence of potential cost overruns, the equilibrium demand for liquidity would be zero.

[28] For tractability, we assume that credit sharing is not possible. In this paper, liquidity can be considered a reduced-form variable that captures all contingency-planning resources spent by the financial entrepreneur.

The first-order conditions for liquidity and borrowing, respectively, are given by:

$$(5.2) \qquad f(l_t^*)(1-l_t^*)\Delta\kappa - \mathrm{E}[\kappa^*] = 0,$$

$$(5.3) \qquad q(1-l_t^*)\mathrm{E}[\kappa^*] = R_t,$$

in which $E[\kappa(l_t)]$ represents the expected units of capital generated per unit of investment and $E[\kappa^*]$ represents its expected level when l_t coincides with its optimal level l_t^*.

Equation (5.2) characterizes the demand for liquidity in the economy and describes the tradeoffs faced by entrepreneurs when choosing liquidity. The first term captures how an increase in liquidity results in a higher likelihood of a successful outcome, but the second term captures how an increase in liquidity causes the scale of investment to fall. This tradeoff is captured in Figure 1. The slope of the marginal benefit curve is given by $f_l(l_t^*)(1-l_t^*)\Delta\kappa - f(l_t^*)\Delta\kappa$, and is guaranteed to be negative as long as $f_l \leq 0$.[29] The slope of the marginal cost curve is given by $f(l_t^*)\Delta\kappa$, which is clearly positive as illustrated in Figure 1.

[Insert Figure 1 Here]

Equation (5.3) characterizes demand for borrowing in this economy. Entrepreneurs are assumed to be price-takers with regard to both the price of capital and the interest rate for borrowing. Since their projects exhibit constant returns to scale, they make zero economic profit and their demand for borrowing is perfectly elastic.[30] As an illustration of how Equation (5.3) determines equilibrium investment, Figure 2 illustrates how the equilibrium investment changes when investors become less willing to borrow due to a reduced collective tolerance to bear losses.

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[29] This assumption will turn out to be a sufficient second-order condition for this benchmark model, and is analogous to assuming that $F(l_t)$ is concave.

[30] To verify this perfect elasticity, note that if households were to demand interest rate $R_t > q(1-l_t^*)E[\kappa^*]$, entrepreneurs would not borrow. If households were to accept interest rate $R_t < q(1-l_t^*)E[\kappa^*]$, entrepreneurs would be willing to undertake investment projects of infinite scale.

The second-order condition for liquidity is:

$$(5.4) \qquad f_l \Delta\kappa(1 - l_t) - 2 * f(l_t)\Delta\kappa < 0.$$

Note that if $f_l \leq 0$, then condition (5.4) is always satisfied. This assumption implies that the probability of a given shock is monotonically decreasing in its magnitude and that the cumulative density function $F(\rho_t)$ is concave in ρ_t. The rest of this section will employ this assumption to ensure that the system is well-behaved.

Finally, assume that following unsuccessful outcomes, entrepreneurs have sufficient income to pay back their loans.[31]

$$(5.5) \qquad \kappa_L(w_t + b_t) \geq E[\kappa^*] * b_t.$$

Assumption (5.5) guarantees that even following a most extreme cost overrun, entrepreneurs have sufficient income to pay off debts, allowing us to study sensitization without introducing credit constraints or similar credit frictions.[32] Satisfying equation (5.5) will require a minimum restriction on w_t relative to b_t but will allow us to demonstrate that sensitization is able to generate procyclical investment even in the absence of credit frictions.[33]

5.2 Model with Sensitization and Loss-Aversion

In this subsection, we augment the model from the last subsection to include sensitization and loss-aversion in order to study sensitization's macroeconomic consequences. Under the gain-loss preferences of Kőszegi and Rabin (2006, 2007), an agent experiences a gain whenever κ_H is realized because the cost overrun is smaller than planned with $\rho_t < l_t$. Gains therefore occur with

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[31] Condition (5.5) guarantees that $q\kappa_L(1 - l_t^*)(w_t + b_t) \geq R_t b_t$ and is derived using Equation (5.3).

[32] As demonstrated by Bernanke and Gertler (1989), the inclusion of these additional concerns would only serve to make investment even more procyclical.

[33] Note that the restriction $w_t \geq \frac{\Delta\kappa}{\kappa_L} b_t$ is sufficient for all distributions $F(l_t)$.

probability $F(l_t)$ and have magnitude $\eta q(\kappa_H - E[\kappa(l_t)])(1-l_t)(w_t+b_t)$.[34] Likewise, losses occur with probability $\left(1-F(l_t)\right)$, and losses are triggered when the cost overrun is higher than expected. When κ_L is realized, the magnitude of the loss is given by $\eta\lambda q(\kappa_L - E[\kappa(l_t)])(1-l_t)(w_t+b_t)$.

Our model of sensitization assumes that past experiences of loss distort the agent's focus from gains to losses. This specification requires that the degree of distortion is proportional to the agent's past experience of loss, $\left(1-\hat{F}(l_{t-1})\right)$. Incorporating the effects of sensitization, her expected utility is therefore:

$$
\begin{aligned}
&E[U(l_t,b_t)] \\
&= q(1-l_t)E[\kappa(l_t)](w_t+b_t) - R_t b_t \\
&+ \eta\lambda \underbrace{\left[1+\alpha\left(1-\hat{F}(l_{t-1})\right)\right]}_{Effect\ of\ sensitization\ on\ loss} \underbrace{*\left(1-F(l_t)\right)}_{prob.\ of\ loss} \underbrace{q(\kappa_L - E[\kappa])(1-l_t)(w_t+b_t)}_{magnitude\ of\ loss} \\
&+ \eta \underbrace{\left[1-\alpha\left(1-\hat{F}(l_{t-1})\right)\right]}_{Effect\ of\ sensitization\ on\ gain} \underbrace{*F(l_t)*}_{prob.\ of\ gain} \underbrace{q(\kappa_H - E[\kappa])(1-l_t)(w_t+b_t)}_{magnitude\ of\ gain}.
\end{aligned}
$$

This expected utility simplifies to:

$$
\begin{aligned}
(5.6)\quad &E[U(l_t,b_t)] \\
&= q(1-l_t)E[\kappa(l_t)](w_t+b_t) - R_t b_t - \eta \\
&* \left[\underbrace{(\lambda-1)}_{loss-aversion} + \underbrace{2\alpha\left(1-\hat{F}(l_{t-1})\right)}_{sensitization} + \underbrace{(\lambda-1)\alpha\left(1-\hat{F}(l_{t-1})\right)}_{interaction\ term}\right] \\
&* qF(l_t)\left(1-F(l_t)\right)\Delta\kappa(1-l_t)(w_t+b_t).
\end{aligned}
$$

If the entrepreneur chooses b_t and l_t optimally, this leads to first-order conditions:

[34] The gain and loss probabilities of this section differ from the gain and loss probabilities from sections 3 and 4 because the risky lotteries of sections 3 and 4 assumed output but not cost uncertainty. As a result, in Sections 3 and 4 gains were realized whenever y was higher than expected. In this section, gains are realized whenever costs are lower than expected, and $F(l_t)$ now refers to the probability of gain.

$$(5.7)\quad R_t = q(1-\widetilde{l_t})E[\kappa(\widetilde{l_t})] - \eta q F(\widetilde{l_t})\left(1-F(\widetilde{l_t})\right)\Delta\kappa(1-\widetilde{l_t}) * \left[(\lambda-1) + 2\alpha\left(1-\hat{F}(l_{t-1})\right) + (\lambda-1)\alpha\left(1-\hat{F}(l_{t-1})\right)\right],$$

$$(5.8)\quad f(\widetilde{l_t})(1-\widetilde{l_t})\Delta\kappa - E[\kappa(\widetilde{l_t})] - \eta\left[(\lambda-1) + 2\alpha\left(1-\hat{F}(l_{t-1})\right) + (\lambda-1)\alpha\left(1-\hat{F}(l_{t-1})\right)\right] * \Delta\kappa\left[\left(1-2F(\widetilde{l_t})\right)(1-\widetilde{l_t})f(\widetilde{l_t}) - F(\widetilde{l_t})\left(1-F(\widetilde{l_t})\right)\right] = 0,$$

in which $\widetilde{l_t}$ represents the second-best liquidity ratio under sensitization and loss-aversion. If $\alpha = 0$ and $\lambda = 1$, these two equations coincide with Equations (5.2) and (5.3). For more general α and λ, Equation (5.7) is a standard (loss-adjusted) zero-profit condition that stems from the simplifying assumption that projects enjoy constant marginal returns in $(w_t + b_t)$.[35] Equation (5.8) represents a standard first-order condition in which there is a tradeoff between liquidity and scale. An increase in liquidity $\widetilde{l_t}$ increases the likelihood that the project will succeed by $f(\widetilde{l_t})$ but reduces the scale of the project by $E[\kappa(\widetilde{l_t})]$. Moreover, increasing $\widetilde{l_t}$ will affect the magnitude of loss, which depends on the term $F(l_t)\left(1-F(l_t)\right)(1-l_t)$. Since the magnitude of $F(l_t)\left(1-F(l_t)\right)(1-l_t)$ is minimized by $\widetilde{l_t} \in \{0,1\}$, the availability and loss-aversion parameters will introduce a tendency toward corner solutions and may result in multiple equilibria in the absence of parameter restrictions forcing a unique interior solution.

For a unique maximum to exist, a sufficient second-order condition is:

[35] This paper's use of the term loss-adjusted profit incorporates both sensitization and loss-aversion, as well as the interaction between the two.

$$(5.9)\quad f_l(\tilde{l_t}) * (1-\tilde{l_t}) - 2f(\tilde{l_t})$$
$$- \eta\left[(\lambda-1) + 2\alpha\left(1-\hat{F}(l_{t-1})\right) + (\lambda-1)\alpha\left(1-\hat{F}(l_{t-1})\right)\right]$$
$$* \left\{f_l(\tilde{l_t})\left(1-2F(\tilde{l_t})\right)(1-\tilde{l_t}) - 2(1-\tilde{l_t})f(\tilde{l_t})^2\right.$$
$$\left.- 2f(\tilde{l_t})\left(1-2F(\tilde{l_t})\right)\right\} < 0.$$

To guarantee a unique $\tilde{l_t}$ to the problem, it is sufficient for the following expression to be negative:

$$f_l(\tilde{l_t}) * (1-\tilde{l_t}) * (1-\eta[(\lambda-1) + 2\alpha + (\lambda-1)\alpha])$$
$$- 2f(\tilde{l_t})\left(1-\eta[(\lambda-1) + 2\alpha + (\lambda-1)\alpha] * \left(1+f(\tilde{l_t})\right)\right) < 0.$$

Since $f_l < 0$ by assumption, then Expression (5.9) is clearly satisfied as long as the loss term $\eta[(\lambda-1) + 2\alpha + (\lambda-1)\alpha]$ is relatively small and $f(0)$ is not too large as captured by the following restriction:

$$(5.10)\qquad \eta[(\lambda-1) + 2\alpha + (\lambda-1)\alpha] < \frac{1}{1+f(0)}.$$

Because $f(0) > 0$, Restriction (5.10) is stricter than the restriction $\eta\lambda < 1/2$ previously introduced for Proposition $1a$. To preclude the possibility of multiple equilibria, the rest of the paper will maintain the stricter Restriction (5.10).

Having discussed the first- and second-order conditions for the entrepreneur's liquidity-scale tradeoff, we now conduct comparative statics to determine the cyclicality of investment and liquidity ratios under sensitization. Starting with investment and interest rate R_t, let $\Lambda_\mathrm{t} = \eta\left[(\lambda-1) + 2\alpha\left(1-\hat{F}(l_{t-1})\right) + (\lambda-1)\alpha\left(1-\hat{F}(l_{t-1})\right)\right]$ represent a comprehensive term of the agent's sensation of net loss, which captures loss-aversion, sensitization, and their interaction. Clearly, Λ_t is increasing in loss-aversion $(\lambda-1)$, the availability parameter α, and the realized frequency of loss when young $\left(1-\hat{F}(l_{t-1})\right)$. Equation (5.11) follows from the Envelope Condition and Equation (5.7):

$$(5.11) \qquad \frac{dR_t}{d\Lambda_t} = -qF(\widetilde{l_t})\left(1 - F(\widetilde{l_t})\right)\Delta\kappa(1 - \widetilde{l_t}) < 0.$$

Equation (5.11) implies that entrepreneurs will require a larger loss-premium when faced with a larger loss term Λ_t, and that this loss-premium will reduce the interest rate they offer to households. If the saving curve is upward-sloping, then households will reduce their quantity of savings. In a closed economy, the investment-savings identity will require that the equilibrium quantity of investment must also fall. These insights are captured by Proposition 2:

Proposition 2. If $S'(R_t) > 0$, then increases in loss-aversion $(\lambda - 1)$, the availability of losses (α), or realized loss when young $\left(1 - \hat{F}(l_{t-1})\right)$ reduce the quantity of investment in a closed economy by leading financial entrepreneurs to offer lower interest rates R_t to households. Furthermore, $\frac{dR_t}{d\left(1-\hat{F}(l_{t-1})\right)} < 0$ suggests that investment is procyclical but that loss premia caused by Λ are countercyclical, with increased realizations of loss leading to decreases in investment but increases in Λ_t.

Proposition 2 states that the availability parameter α responsible for sensitization reinforces the distortions in investment caused by loss-aversion and that both sensitization and loss-aversion act to reduce investment by increasing loss-premia. Proposition 2 also suggests that the dynamics of loss premia are countercyclical but that the dynamics of investment are procyclical because realized losses rise during recessions, causing Λ_t to increase and the quantity of investment to fall. A graphical representation of Proposition 2 is presented in Figure 2. The zero-profit condition of Equation (5.7) implies that investment is perfectly elastic in R_t. In Figure 2, an increase in Λ from Λ_t to $\Lambda_t{}'$ following an increase in $\left(1 - \hat{F}(l_{t-1})\right)$ will cause the investment schedule to shift down as $\frac{dR_t}{d\Lambda_t} < 0$. As long as the savings function is upward-sloping, an increase in the loss premium Λ_t will result in a decrease in the economy's equilibrium level of investment as shown in Figure 2.

[Insert Figure 2 Here]

Relative to its effects on R_t, the effects of an increase in Λ_t on $\widetilde{l_t}$ are more ambiguous. From Equation (5.6), the agent's sensation of net loss depends positively on $(1-l_t)F(l_t)\big(1-F(l_t)\big)$, which is minimized for $l_t \in \{0,1\}$. From Equation (5.8),

$$(5.12) \quad \frac{d\widetilde{l_t}}{d\Lambda_t} = \Delta\kappa \frac{\left[\left(1-2F(\widetilde{l_t})\right)(1-\widetilde{l_t})f(\widetilde{l_t}) - F(\widetilde{l_t})\left(1-F(\widetilde{l_t})\right)\right]}{(1-\widetilde{l_t})\left(f_l(\widetilde{l_t}) - \frac{2f(\widetilde{l_t})}{(1-\widetilde{l_t})} - \Lambda_t\left(1-2F(\widetilde{l_t})\right)\left\{f_l(\widetilde{l_t}) - \frac{2f(\widetilde{l_t})^2}{1-2F(\widetilde{l_t})} - \frac{2f(\widetilde{l_t})}{(1-\widetilde{l_t})}\right\}\right)},$$

with the denominator less than zero when Restriction (5.10) holds and the numerator equal to the slope of $(1-l_t)F(l_t)\big(1-F(l_t)\big)$. If the liquidity ratio l^* from Equation (5.2) exceeds the liquidity ratio maximizing $F(l)\big(1-F(l)\big)(1-l)$, then $\Lambda_t > 0$ will encourage the agent to reduce the sensation of net loss by increasing the liquidity ratio. If the liquidity ratio l^* from Equation (5.2) is below the liquidity ratio maximizing $F(l)\big(1-F(l)\big)(1-l)$, then $\Lambda_t > 0$ will encourage the agent to reduce the sensation of net loss Λ_t by decreasing the liquidity ratio. The following proposition captures the economic intuition behind Equation (5.12):

Proposition 3. Increases in loss-aversion $(\lambda-1)$*, the availability of loss* (α)*, or realized loss when young* $\left(1-\hat{F}(l_{t-1})\right)$ *distort the liquidity ratio from its first-best level toward extreme values,* $l_t \in \{0,1\}$*. If loss is relatively unlikely with* $F(\widetilde{l_t}) \approx 1$*, then increases in loss-aversion, the availability of loss, and past experience of loss will increase the agent's liquidity ratio. If loss is relatively* *likely with* $F(\widetilde{l_t}) \approx 0$*, then increases in loss-aversion, the availability of loss, and past experience of loss will decrease the agent's liquidity ratio.*

Proposition 3 suggests that *distortions* in liquidity ratios are countercyclical but that liquidity ratios may be either countercyclical or procyclical. Although this ambiguous theoretical finding contrasts with Kato's (2006) empirical finding of countercyclical liquidity ratios, to the extent that the probabilities of loss are generally low in a normal economy, Proposition 3 would suggest that liquidity ratios should be countercyclical.

6. Concluding Comments

In this paper, we developed preferences consistent with long-run sensitization based on our understanding of Kandel's (2001) discussion of sensitization, Kahneman's (2011) discussion of the availability heuristic, and our understanding of the psychological phenomenon of extraordinary persistence of first-time memories from young adulthood referred to as the reminiscence bump. We believe that this paper's model of long-run sensitization may explain the long-run changes in risk preferences documented in the empirical literature, with past experiences of loss increasing the loss-premium required by agents in the model. To show that these preferences are well-behaved, sensitization is shown to preserve consistency with stochastic dominance.

One of the limitations of the model of sensitization is that sensitization, like loss-aversion, may induce agents to prefer corner solutions. In addition, sensitization's effects at the intensive margin (e.g., liquidity) may be indeterminate and depend on the probability of loss. When loss is relatively unlikely, increases in sensitization and loss-aversion induce the agent to decrease her overall sensation of net loss by increasing liquidity. When loss is relatively likely, increases in sensitization and loss-aversion induce the agent to decrease her overall sensation of net loss by decreasing liquidity.

In conclusion, we note that the results for loss-aversion and sensitization starkly contrast with conventional models of risk-aversion. Conventional models of risk-aversion suggest that increases in risk-aversion reduce risk-taking on both the extensive and intensive margins. In contrast, this paper finds that increases in loss-aversion (as well as sensitization) reduce risk-taking on the extensive margin but may *increase* risk-taking on the intensive margin when the probability of loss is high. We believe that our findings suggest the need for further research to better understand the macroeconomic consequences of sensitization and loss-aversion.

Appendix: Proofs Related to First-Order Stochastic Dominance

Proof of Proposition 0a:

Suppose that lottery F exhibits strict first-order stochastic dominance over G. We wish to show that the expected utility of F, denoted $E[U(\hat{y})|F]$, will exceed the expected utility of G. From Equation $(3.2b)$, the expected utility for lottery F is:

$$E[U(\hat{y})|F] = \mu_F + \eta\lambda \int_{-\infty}^{\mu_F} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y} + \eta \int_{\mu_F}^{\infty} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y}.$$

Adding and subtracting $\eta \int_{-\infty}^{\mu_F} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y}$ to the right-hand side yields:

$$E[U(\hat{y})|F] = \mu_F + \eta\lambda \int_{-\infty}^{\mu_F} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y} - \eta \int_{-\infty}^{\mu_F} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y}$$
$$+ \eta \int_{-\infty}^{\infty} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y}.$$

Since $\int_{-\infty}^{\infty} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y} = 0$, the above expression simplifies to:

$$E[U(\hat{y})|F] = \mu_F + \eta(\lambda - 1) \int_{-\infty}^{\mu_F} (\hat{y} - \mu_F) f(\hat{y}) d\hat{y}.$$

Integration by parts yields:

$$E[U(\hat{y})|F] = \mu_F + \eta(\lambda - 1) * \left((\hat{y} - \mu_F) * F(\hat{y})|_{-\infty}^{\mu_F} - \int_{-\infty}^{\mu_F} F(\hat{y}) d\hat{y} \right).$$

Since F's support is bounded below, $\lim_{\hat{y}\to-\infty} \hat{y} * F(\hat{y}) = 0$, and the above expression simplifies to:

$$(A1) \qquad E[U(\hat{y})|F] = \mu_F - \eta(\lambda-1) * \int_{-\infty}^{\mu_F} F(\hat{y})d\hat{y}.$$

We use the same reasoning to obtain the analogous expression for the expected utility associated with choosing lottery G:

$$(A2) \qquad E[U(\hat{y})|G] = \mu_G - \eta(\lambda-1) * \int_{-\infty}^{\mu_G} G(\hat{y})d\hat{y}.$$

We now use Equations (A1) and (A2) to show that lottery F must be associated with greater expected utility according to the definition of first-order stochastic dominance, which will complete the proof. According to the definition of first-order stochastic dominance (see Mas-Colell, Whinston, and Green (1995)), $F(\hat{y}) \le G(\hat{y})$ for each possible realization $\hat{y}$. According to strict dominance, $\mu_F > \mu_G$. Then Equations (A1) and (A2) directly imply that:

$$E[U(\hat{y})|F] - E[U(\hat{y})|G] = (\mu_F - \mu_G) - \eta(\lambda-1) * \left(\int_{\mu_G}^{\mu_F} F(\hat{y})d\hat{y} + \int_{-\infty}^{\mu_G} F(\hat{y})d\hat{y}\right) + \eta(\lambda-1)\int_{-\infty}^{\mu_G} G(\hat{y})d\hat{y}.$$

If the above expression is strictly positive under strict dominance, then this will complete the proof. Since $\eta(\lambda-1) * \left(\int_{-\infty}^{\mu_G} F(\hat{y})d\hat{y}\right) \le \eta(\lambda-1) * \left(\int_{-\infty}^{\mu_G} G(\hat{y})d\hat{y}\right)$ by definition, then to prove Proposition 0a, it is only necessary to show that:

$$\mu_F - \mu_G > \eta(\lambda-1) * \left(\int_{\mu_G}^{\mu_F} F(\hat{y})d\hat{y}\right).$$

To prove this desired relationship, note that if $\eta(\lambda-1) < 1$, then:

$$\mu_F - \mu_G = \int_{\mu_G}^{\mu_F} 1 * d\hat{y} \ge \int_{\mu_G}^{\mu_F} F(\hat{y})d\hat{y} > \eta(\lambda-1)\int_{\mu_G}^{\mu_F} F(\hat{y})d\hat{y}\,. \blacksquare$$

Proof of Proposition 1a:

Suppose that lottery F exhibits strict first-order stochastic dominance over G. We will show that if $\eta\lambda < 1/2$, then the expected utility associated with lottery F

strictly exceeds the expected utility associated with G, $E[U(y_{t+1})|F] > E[U(y_{t+1})|G]$.

From Equation (4.1), the expected utility for lottery F is:

$$E[U(y_{t+1})|F] = \mu_F + \eta\left(1 - \alpha\widehat{H}_t(\mu_H)\right)\int_{\mu_F}^{\infty}(y_{t+1} - \mu_F)f(y)dy$$
$$+ \eta\lambda\left(1 + \alpha\widehat{H}_t(\mu_H)\right)\int_{-\infty}^{\mu_F}(y_{t+1} - \mu_F)f(y)dy.$$

Following the sequence of steps in the proof of Proposition 0a previously in the appendix, this expected utility can be written in the following form:

$$(A3) \qquad E[U(y_{t+1})|F] = \mu_F - \eta\left((\lambda - 1) + \alpha(\lambda + 1)\widehat{H}_t(\mu_H)\right)\int_{-\infty}^{\mu_F}F(y)dy.$$

We can then provide a parallel expression for the expected utility of lottery G:

$$(A4) \qquad E[U(y_{t+1})|G] = \mu_G - \eta\left((\lambda - 1) + \alpha(\lambda + 1)\widehat{H}_t(\mu_H)\right)\int_{-\infty}^{\mu_G}G(y)dy.$$

The difference between the expected utilities is captured by

$$(A5) \quad E[U(y_{t+1})|F] - E[U(y_{t+1})|G]$$
$$= \mu_F - \mu_G$$
$$- \eta\left((\lambda - 1) + \alpha(\lambda + 1)\widehat{H}_t(\mu_H)\right)\left(\int_{-\infty}^{\mu_F}F(y)dy - \int_{-\infty}^{\mu_G}G(y)dy\right).$$

If $\alpha\widehat{H}_t(\mu_H) = 0$, then it follows from Proposition 0a that $E\big(U(y_{t+1}|F)\big) - E\big(U(y_{t+1}|G)\big) > 0$. Then note that $E\big(U(y_{t+1}|F)\big) - E\big(U(y_{t+1}|G)\big)$ is monotonic with respect to $\alpha\widehat{H}_t(\mu_H) \in [0,1]$ with

$$\frac{d(E[U(y_{t+1})|F] - E[U(y_{t+1})|G])}{d\left(\alpha\widehat{H}_t(\mu_H)\right)} = \eta(\lambda + 1)\left(\int_{-\infty}^{\mu_G}G(y)dy - \int_{-\infty}^{\mu_F}F(y)dy\right),$$

$$\frac{d^2(E[U(y_{t+1})|F] - E[U(y_{t+1})|G])}{d\left(\alpha\widehat{H}_t(\mu_H)\right)^2} = 0.$$

As a result, if $E[U(y_{t+1})|F] - E[U(y_{t+1})|G] > 0$ under the most extreme form of sensitization, $\alpha\widehat{H}_t(\mu_H) = 1$, then $E[U(y_{t+1})|F] - E[U(y_{t+1})|G] > 0$ will hold true for all intermediate levels of sensitization $\alpha\widehat{H}_t(\mu_H) \in (0,1)$.

For the remainder of this proof, suppose that sensitization takes on its most extreme form possible, with $\alpha\widehat{H}_t(\mu_H) = 1$. Then given $\int_{\mu_G}^{\mu_F} F(y)dy \leq \mu_F - \mu_G$ and given Equation $(A5)$:

$$(A6) \qquad E[U(y_{t+1})|F] - E[U(y_{t+1})|G]$$
$$\geq (\mu_F - \mu_G)(1 - 2\eta\lambda) + 2\eta\lambda\left(\int_{-\infty}^{\mu_G} G(y)dy - \int_{-\infty}^{\mu_G} F(y)dy\right).$$

Since (strict) first-order stochastic dominance guarantees that $(\mu_F - \mu_G) > 0$ and that $\int_{-\infty}^{\mu_G} G(y)dy - \int_{-\infty}^{\mu_G} F(y)dy \geq 0$, we conclude that sensitization is consistent with first-order stochastic dominance under the parameter restriction $\eta\lambda < 1/2$. ■

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Figures

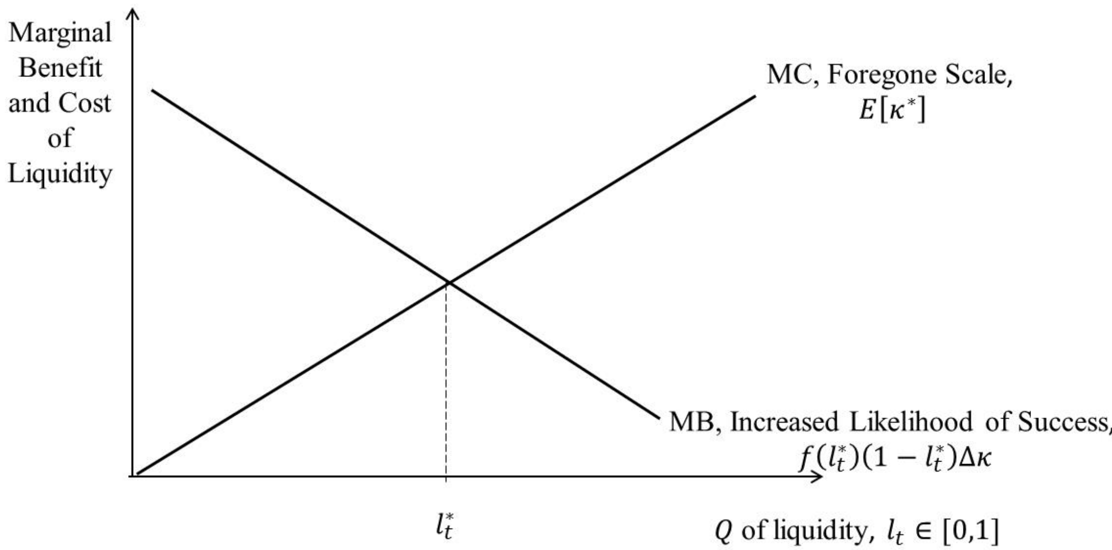

FIGURE 1. THE LIQUIDITY-SCALE TRADEOFF

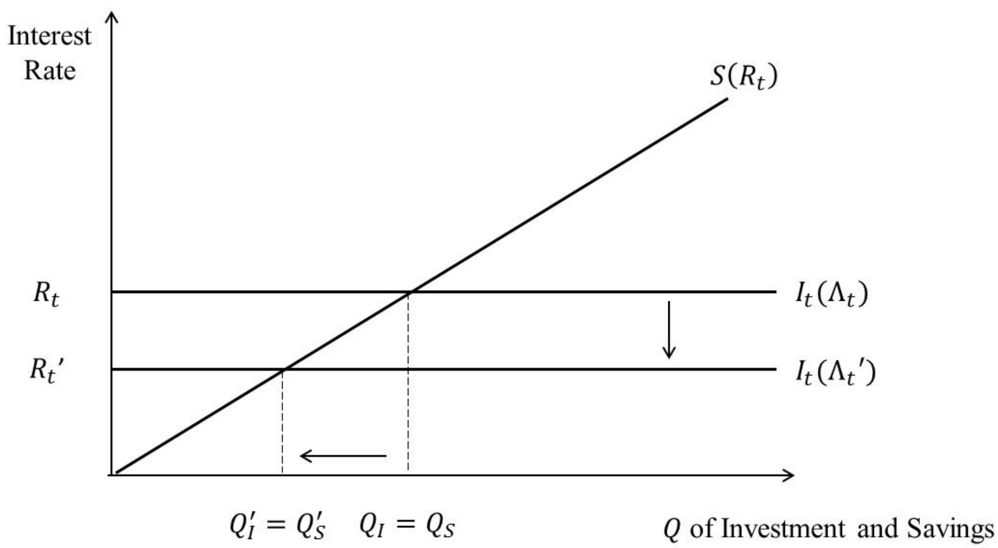

FIGURE 2. EQUILIBRIUM INTEREST RATES AND INVESTMENT WHEN AGENTS BECOME SENSITIZED BY PRIOR LOSS