Promising methods, questionable results: Developing an inverse model of the fossil ridge-transform intersection in the Troodos ophiolite in Cyprus

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Senior Integrative Exercise
March 13, 2019

Submitted in partial fulfillment of the requirements for a Bachelor of Arts degree from Carleton College, Northfield, Minnesota.
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ABSTRACT

Forward models are commonly used to investigate the stress regime of ridge-transform intersections. However, none of these previous studies have used real data to create an inverse model. I used forward modeling and a maximum likelihood estimate to create an inverse model of the fossil ridge-transform intersection in the Troodos ophiolite in Cyprus. I use the forward model to predict the spreading rate, rate of left lateral slip, and thermal properties based on observed dike pole directions. With this analysis, I find a half-spreading rate of 101.2 mm/yr and a slip rate 10.3 mm/yr best fit the observed dike pole orientations. The ratio of slip to spreading rate is much lower than predicted by previous forward models of ridge-transform intersections. However, the predicted orientations in the best case are still significantly different from the observed orientations, which indicates that there are likely problems with the model, its implementation, and perhaps with its assumptions about the behavior of dikes themselves.

Keywords: Ridge-Transform Intersection, Inverse Model, Cyprus, Maximum Likelihood Estimate
INTRODUCTION

There are two common approaches for modeling geologic systems: forward modeling and inverse modeling (Pollard and Fletcher, 2005). Forward models use a set of parameters and initial conditions to predict the deformation caused by those parameters (Pollard and Fletcher, 2005). Forward models are often not connected to real world observations in any concrete way, which can make it difficult to validate their results. By contrast, inverse modeling uses field data to directly predict the conditions producing those data (Pollard and Fletcher, 2005). These models are grounded in real data but are difficult to formulate. For the tectonic system of ridge-transform intersections, forward models are typically used.

There have been numerous attempts to forward model a ridge-transform intersection. Several early workers studied stresses surrounding ridge-transform intersections in 2-D (Fujita and Sleep, 1978; Phipps Morgan and Parmentier, 1984). More recently, the models are 3D for either stresses or velocities (Grindlay and Fox, 1993; Furlong et al. 2001; Behn et al., 2002). These studies often compare their results qualitatively with maps of structures at ridge-transform intersections, but none incorporate actual field data in their analysis.

Inverse models are not often used to study ridge-transform intersections. In this study, I build an inverse model for the Troodos ophiolite in Cyprus. The field data include thousands of measurements of dikes. I build on computation framework of the inverse model developed in Bacon (2017). In this paper, I present an extensive geologic and mathematic background to provide the basis for the model and analysis. I then explain implementation of the model and various maximum likelihood estimates. Finally, I discuss the results of this analysis along with the drawbacks to my methods and the models.
GEOLOGIC BACKGROUND

Mid-ocean ridges are composed of offset ridge segments, which are connected by transform faults. Ridge-transform intersections occur where midocean spreading ridge plate boundaries meet transform faults (Fig. 1). Although these intersections are found along all mid-ocean ridges, their characteristics vary depending on spreading rate (Gerya, 2012). Slow ridges, with a half-spreading < 25 mm/yr, exhibit significant thinning in the crust at the transforms while fast ridges, with a half-spreading rate of > 40 mm/yr, do not show as much thinning (Gerya, 2012). The transform faults often exhibit slow steady slip instead of fast ruptures (Boettcher and Jordan, 2004). The thermal structure of ridge-transform systems is also complex. The thermal structure of spreading ridges is relatively well-understood, but the heat conduction across ridge-transforms makes the temperature more complicated (Furlong et al, 2001).

The features local curve of the ridge towards the transform in both slow and fast spreading ridges (Phipps Morgan and Parmentier, 1984). However, extensional features in the ridge-transform intersection take two different forms: J-shaped and anti-J-shaped (Sonder and Pockalny, 1999). Figure 1 shows an example of a J-shaped curvature to the normal faults which means that these features curve with the ridge. In an anti-J-shaped ridge-transform, the extensional features curve in the opposite direction or the ridge (Sonder and Pockalny, 1999).

Cyprus

The island of Cyprus is in the Mediterranean and contains the Troodos ophiolite, which consists of a full sequence of oceanic crustal rocks from ultramafic rocks to pillow basalts (Fig. 2; Moores and Vine, 1971). The ophiolite formed in the Late Cretaceous above a
Figure 1. A right lateral mid-ocean ridge-transform in the Pacific ocean with the major features highlighted. Spreading ridges and the transform fault are indicated in red and the motions are black arrows. The inside corners are indicated with red highlights and the extensional features are outlined in black. The extensional figures are parallel to the ridge then curve towards the transform near the transform making this a J-shaped ridge-transform intersection. The orientations of these features are then deformed by shearing along the fault.
subduction zone and shows little to no deformation from its emplacement on Cyprus (Moores et al., 1984).

There are three fossil ridge segments exposed in the ophiolite: the Solea, Mitsero, and Larnaca grabens. The N-S striking Solea graben is thought to have been the most prominent ridge when spreading was occurring (Varga and Moores., 1985). There is evidence of a fossil transform fault called the Arakapas fault perpendicular to the ridge in the Troodos ophiolite (Simonian and Gass, 1978). The fault is characterized by highly deformed rocks and a topographic low (Simonian and Gass, 1978). To the north of the fault, sheeted dikes have a N strike that curves to the east the closer the dikes are to the fault. The orientation of the dikes has been used to determine motion along the Arakapas fault as dextral (Bonhommet et al., 1989), which implies that the ophiolite was formed at a ridge transform intersection (Scott et al., 2013).

MATHEMATIC BACKGROUND

Before going into the specifics of the model that I am using, I must give some background on the math behind forward and inverse models. First, I will examine the past literature on forward modeling of a ridge-transform intersection, then I will show the governing partial differential for the case of an elastic solid. I will also discuss the math used to find the predicted orientations of dikes using the results of the finite element analysis. Finally, I will discuss the statistics I am using to calculate a maximum likelihood estimate for the parameters.

Modeling

Numerical models of stresses around ridge-transform intersections laid the groundwork for this project. The earliest study by Fujita and Sleep (1978) used a 2D finite
Figure 2. Map of the Troodos Ophiolite in Cyprus with the rock type plotted and the Solea graben and Arakapas fault in red on the map. Mild doming in the center of the map combined with erosion allows the entire ophiolite sequence to be exposed.
element model that applied forces on both sides of the ridge and an anisotropic material (Fig. 3A). Phipps Morgan and Parmentier (1984) used 2D finite element modeling techniques to apply stresses to the ridge and transform resisting spreading. They applied traction boundary conditions to the ridges and the transform to represent resistance to spreading. The resulting stresses matched features observed in ridge-transform intersections (Fig. 3B) and produced more variation in the stress direction than Fujita and Sleep (1978). The study made it clear that the orientation of stresses strongly relies on the boundary conditions of the study—in particular the ratio of force on the transform to force on the ridges. Grindlay and Fox (1993) worked to apply the methods of Phipps Morgan and Parmentier (1984) to different ridge-transform geometries. They also added in a temperature component, which resulted with the stresses in Figure 3C. Behn et al. (2002) used 3D finite element analysis to demonstrate that transform faults act as zones of weakness and a coupling of around 5% explains the observed patterns in deformation (Fig. 3D). Neves et al. (2004) created a 2D finite element model that arrived at a different result than many of the other studies based on topographical features and their effects on the stress field as is seen in Figure 3E. Overall, most models have the maximum compressive stress in similar directions as summarized in Figure 3F.

Other models have incorporated the thermal structure surrounding ridge-transform intersections. Furlong et al. (2001) used a 3D finite element mesh with a simple geometry coupled with a separate thermal model. Together, Furlong et al. ran the models back and forth until they reached a steady state solution. This model was focused on velocity structure and thermal properties rather than stresses.

**Continuum Mechanics**

Running a forward model requires equations to describe physical processes. Since the goal is to run an inverse model using a forward model, the forward model must produce a
Figure 3. The maximum horizontal stress directions in five past studies and a summary of what the papers have in common (Scott et al., 2013). (A) The stress directions from Fujita and Sleep (1978). (B) The stress directions from Phipps Morgan and Parmentier (1984). (C) The stress directions from Grindlay and Fox (1993). (D) The stress directions from Behn et al. (2002). (E) The stress directions from Neves et al. (2004). (F) Demonstrates the commonality between various models. Most models have ridge parallel maximum horizontal compressive stress direction near the ridge and the inside corner always have stresses striking to the NW.
velocity field and a stress tensor field. These can be determined with Navier’s equations for elastic materials derived from Cauchy’s momentum equation.

The general form Cauchy’s momentum equation is:

$$\rho \frac{Dv}{Dt} = \nabla \cdot \sigma + \rho g,$$

where $\rho$ is density, $v$ is the velocity column vector, $\nabla$ is the row vector $\left[ \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3} \right]$, $\sigma$ is the stress tensor, and $g$ is the vector of gravity. Cauchy’s momentum equation is the primary differential equation that is used to determine deformation and motion in a complex system.

Like most of the previous models, I will assume the oceanic crust behaves as an elastic material, which means that the Cauchy’s momentum equation can be put into a less general form called Navier’s Equations:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \mu \nabla^2 u_i + (\mu + \lambda) \frac{\partial}{\partial x_i} (\nabla \cdot u) + \rho g_i,$$

where $u$ is the displacement vector and $\mu$ and $\lambda$ are Lamé’s Constants, which describe the material properties of the rock. Navier’s Equations are complex formulae that, when given boundary conditions, provide a displacement field and a stress tensor field. Unlike other forms of Cauchy’s momentum equation (such as Navier-Stokes equation for viscous materials), there is no time component on the right side.

**Temperature Relationship to Young’s Modulus**

Temperature is a confounding factor in the development and deformation in a ridge-transform intersection and should be included in a 3D forward model. First, I will describe the equations behind normal oceanic crust. After that, I will relate temperature to the Young’s Modulus of the material.
I use a fairly simple equation for the temperature of the oceanic crust derived from the differential equation:

\[ \frac{\kappa \partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}, \]

where \( \kappa \) is the thermal diffusivity, \( T \) is temperature, \( z \) is depth, and \( t \) is time (Turcotte and Schubert, 2002). I want an equation in the form \( T(z,t) \) and I can find this using boundary cases: \( T(0,t) = T_s \) and \( T(z,t) = T_m \) where \( T_s \) is the surface temperature of the oceanic crust and \( T_m \) is the melting temperature of the basalt. Using these boundary conditions, I can obtain the equation:

\[ T(z,t) = T_s + (T_m - T_s) \text{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right), \]

where \( \text{erf} \) is the error function (Turcotte and Schubert, 2005). This equation is the temperature structure of ordinary oceanic crust. However, when in a ridge-transform intersection, there is a lot more interaction that must be considered due to the offset of the ridges and the conductivity across the transform fault. The implementation will be further explored in the model implementation section.

The second part of this computation is a much less well-defined relationship between Young’s Modulus and temperature. Broadly speaking, Young’s Modulus decreases as temperature increases, meaning that, at higher temperatures, less stress is needed to stretch a material a given amount. A basic general form of this relationship from Courtney (2005) is:

\[ E = E_0 \left(1 - \alpha \frac{T}{T_m}\right), \]

where \( E \) is Young’s Modulus, \( E_0 \) is Young’s Modulus at 0 K, \( \alpha \) is a fitting constant, and \( T_m \) is the melting temperature. The equation is a very simplified form of a more complex relationship.
Prediction of Data

The main dataset I am using for this model is dike data from the Troodos ophiolite in Cyprus. The dike locations are shown in Figure 4. Using the forward model described by the equations above, I make predictions about the formation and subsequent deformation of dikes based on several assumptions. First, I assume that dikes form perpendicular to the least compressive stress. Second, I assume that dikes deform passively, which means that the dike has the same competency than the rock around it, causing it to deform passively with the rock around it. I determine the least compressive stress direction using the eigenvector with the greatest eigenvalue of the stretching tensor \( \mathbf{D} \), which I calculate from the velocity gradient tensor \( \mathbf{L} \) using the equation \( \mathbf{D} = \frac{1}{2} (\mathbf{L} + \mathbf{L}^T) \) (Davis and Titus, 2011). Therefore, using the velocity gradient tensor where a dike is formed, I am able to calculate its initial pole \( \mathbf{p}(0) \). So, if I have a current location for the dike, a velocity field, and velocity gradient tensor field, and an age, I can trace the dike back through the velocity field to the location where it was formed and find its initial pole. I can then solve for the pole of the dike at the current time (time 1) using the finite deformation tensor \( \mathbf{F} = \exp(\mathbf{L}) \) with the equation:

\[
\mathbf{p}(1) = \frac{(\mathbf{F}^{-1})^T \mathbf{p}(0)}{|(\mathbf{F}^{-1})^T \mathbf{p}(0)|} \quad \text{(Davis and Titus, 2011)}.
\]

Statistics

In this section, I will outline the statistics I used to use a forward model to design an inverse model. These methods will also help validate the model and inform the quality of the forward model at modeling the geologic system. First, I will discuss a maximum likelihood estimate and second, I will discuss the likelihood estimate for the data.

The forward model is able to use boundary conditions to predict the final orientation for observed dikes. Specific probability distributions are able to compare the predicted dike
Figure 4. A map of the Troodos ophiolite with the Solea graben and the Arakapas fault labeled in red and all of the collected dike data plotted as points on the ophiolite (Titus et al., 2016). The points used for this analysis are to the east of the Solea graben which includes 2298 dike pole directions.
poles to the observed dike poles to compute a likelihood of the boundary conditions derived from parameters. In a general sense, with a probability density function of \( f(X) \), parameters \( \theta \), and data from \( X = \{x_1, \ldots, x_n\} \), the likelihood is

\[
L(\theta|x_1, ..., x_n) = \prod_{i=1}^{n} f(x_i|\theta).
\]

Finding the input of parameters \( \theta \) with the global maximum likelihood is called a maximum likelihood estimation.

The data that I am using for the maximum likelihood analysis are dike pole data. To find the likelihood function, I must have a probability density function for dike pole data. Noise in the data is an important factor in the likelihood calculation. A predicted dike pole can be compared to an observed dike pole using the smallest angle between a predicted dike pole and an observed dike pole. I use the Watson distribution around the observed dike pole \( p_o \). Plugging a prediction into the distribution results in a likelihood:

\[
f(p_o|p, \kappa) = \frac{\sqrt{\kappa}}{e^{\kappa F(\sqrt{\kappa})}} \exp(\kappa(p_o^T p)^2),
\]

where \( \kappa \) is the concentration parameter and \( F \) is the Dawson function (Mardia and Jupp, 1972). The concentration parameter acts as the amount of noise expected in the data. The Watson distribution results in an overall likelihood that we would observe the measured dike poles, given the poles predicted by the forward model. So, to find the overall likelihood of the observed poles given the predicted poles based on the forward model is the product of the results of the Watson distribution. To make this computation easier, I use the sum of the log likelihood. With all these components, I designed a forward model, use it to predict data, and compare the predictions to the observed values to find a likelihood.
IMPLEMENTATION

The methods I use are designed to combine a rigorous numerical model and real data within a statistical format. The model constructed for this study uses the differential equation solver PyLith and a finite element mesh used to represent a ridge-transform intersection to compute a velocity field and a stress tensor field. The model is called and processed by code originally described in Bacon (2017) with adaptions and additions that I made. In this section I will talk the partial differential equations solver PyLith, the Finite Element Meshes I considered for the model, and finally the code to run the MLE.

Finite Element Analysis

Finite element analysis allows us to solve equations in a volume without solving them over the continuous volume by imposing a mesh on the volume and then solving the equations for each point in the mesh. Solving equations on a mesh reduces the problem to solving a finite number of equations in the form of Cauchy’s Momentum Equation, shown above. The model accomplishes solves Cauchy’s momentum equation using the boundary conditions provided by providing displacements on the sides relative to the spreading rate and the slip on the transform fault. After the program solves the equations over the finite element mesh, I am able to predict the values at selected points (the stations where data was collected) by interpolating over the grid of solved points.

There were two geometries and boundary conditions that I tested in preliminary model runs. The first model was created initially by Bacon (2017) based on the model from Phipps Morgan and Parmentier (1984). This model contains only one side of the ridge-transform system (Fig. 5A). I apply normal traction on the ridge and a tangential traction on the transform. The second model was adapted from the model in Furlong et al. (2001). This
Figure 5. A comparison of the two main models that I investigated: the Traction model (A) and the Displacement model (B). Solid arrows represent displacement boundary conditions and hollow arrows represent traction boundary conditions (forces). The thick black line in A represents a fixed displacement boundary condition.
model which includes two segments of oceanic crust separated by a transform fault creating two inside corners rather than just one (Fig. 5B). Here, the boundary conditions are displacement conditions on the north and south faces that are proportional to the spreading rate and a displacement condition on the transform fault that is proportional to slip rate.

Initial runs of the traction model produce a stress tensor field similar to those from Phipps Morgan and Parmentier (1984) (Fig. 6). However, due to the model being treated as an elastic material by PyLith in order to reduce computation time for the maximum likelihood estimates, I was unable to convert the results to a velocity field, which is required later for analysis of field data.

The shortcoming of the traction model led me to develop the displacement model, which uses displacement boundary conditions to the north and south that mimic spreading rates. By converting the displacement conditions on the boundaries to spreading rates, I am able to extract a velocity field from the displacement field output. Because this version provides a velocity field, I use it for later model runs.

Next, I incorporate a temperature model of a mid-ocean ridge (Fig. 7). I used the relationship between temperature and Young’s Modulus defined in the temperature section to attempt to incorporate the effects of temperature into the model by changing the Young’s Modulus. I used a simplified version of the final temperature model from Furlong et al. (2001) to describe regions where temperature behaved like regular oceanic crust. Within 5 km of the transform fault, I took a weighted average of the temperatures on both sides of the transform fault based on the position to the fault. I then convert the temperatures to Young’s Modulus. I converted these numbers into $V_s$ and $V_p$ and input them into PyLith as material constants at each point.
Figure 6. Plotted is the maximum horizontal compressive stress directions from the traction model (A) compared to the maximum horizontal compressive stress direction based on previous Phipps Morgan and Parmentier (1984) model (B). The scale of the models is slightly different with a 60 km fault in (A) and a 100 km fault in (B). The stress directions are very close to the same. Near the ridge is the only place where the model deviates from the Phipps Morgan and Parmentier (1984) model as the stresses curve less towards the transform. In general, the model does a good job of replicating stress directions.
Figure 7. (A) The temperature structure for the Displacement model at 10 km depth based on the equations in the temperatures relationship to Young’s Modulus section. (B) The temperature structure for the Displacement model at 5 km depth. The length of the transform is 60 km and the length of each ridge segment is 25 km. The temperature interacts across the fault within 10 km of the fault.
Combining the geometry and boundary conditions of the displacement model with the thermal approach, the parameters in the model are half spreading rate, slip rate, the fitting constant $\alpha$, and thermal diffusivity. After I set up the model, I used the program PyLith to solve the differential equations discussed in the continuum mechanics which outputs a velocity field (dependent on the change in time given) and a stress tensor field. These outputs are applied over the mesh of the model, which splits the model into a finite number of points. I was then able to take the outputs on those points and extrapolate the velocity field and the stress tensor field to any point in the grid.

**Optimization Implementation**

To incorporate field data into the modeling process, I use the model results as follows. First, the code writes PyLith files based on the parameters and runs PyLith on the model, which outputs velocity and stress tensor fields (Fig. 8B). Then, the code takes the locations of the data (Fig 8A) and computes their trajectories backwards in time using the velocity field to their initial locations (Fig. 8C). The initial locations are defined by a certain total time (1 myr) or reaching the edge of the mesh (the ridge). Once the initial location for each dike is calculated, the initial orientation is calculated (Fig. 8D), and the observed dike pole is predicted using the finite deformation tensor (Fig. 8E). I use these predictions to calculate the log likelihood for the initial input of parameters (Fig. 9). Then I ran the code through an optimizer to change the parameters until the likelihood is maximized, which provides the framework for the rest of this project. Given data and starting parameters, the code returns the best parameters to fit the data (Fig. 8F) based on the model I am using.

After I designed the model and finalized the code, I began to run simulations with synthetic data. The program takes less time to run if the initial parameters are close to the
Figure 8. Cartoon illustrating the process of predicting dike pole directions from a velocity field and stress tensor field. First, observed data points are plotted with their observed dike pole orientation (A). Next, the velocity field is plotted (B). Third, the position of the dikes is traced back through the velocity field for a specified amount of time using the Runge-Kutta method (C). Then, the predicted initial dike orientation is predicted using the least compressive stress direction (D). In (E), the rotation of the dikes is calculated along the path using the velocity gradient tensor field which is calculated from the velocity field. Finally, the predicted dike pole directions are compared to the observed dike pole directions (F).
Figure 9. A flow chart demonstrating the execution of the code behind the Maximum Likelihood Estimate. Parameters are input which writes the PyLith files to design and implement the Finite Element Analysis. The code predicts the expected dike orientation at each location with observed data. The observed and predicted dike poles are then compared to obtain a likelihood. New parameters are chosen and the code is run until it reaches a maximum likelihood.
true answer. Additionally, starting parameters that are too far from the final answer can optimize to false maximums.

RESULTS

Based on the results of the synthetic data, I first ran a program on 23 of the 2298 dike pole measurements i.e. one-one-hundredth of the full dataset, which ran the maximizer with random starting parameters to find the parameters that produced the highest log likelihood. Each run of the maximum likelihood estimate for the one-one-hundredth dataset took about half of a day. I then took the result with the highest likelihood and used it as the initial parameters for the full run of the maximum likelihood estimate with 2298 dike pole measurements. Based on these initial runs, I used initial parameters of 81 mm/yr half spreading rate, 7 mm/yr slip rate (right-lateral slip), $\alpha = 0.72$, and 57.8 thermal diffusivity. The MLE for the full 2298 dike poles ran for about 10 days. In this section I will analyze the results of the small dataset and the full data set.

Small Dataset

The first analysis I ran was on 23 of the 2298 data points. The likelihood estimate was run ~30,000 times, which is enough data to contour the log likelihood based on various parameters (Fig. 10). At the time of the beginning of the run of the full dataset, the highest likelihood from this analysis came from the parameters 81 mm/yr half spreading rate, 7 mm/yr slip rate (right-lateral slip), $\alpha = 0.72$, and 57.8 thermal diffusivity. However, subsequent results produced a different highest likelihood with a right-lateral slip rate on the fault and a smaller spreading rate, a geologically impossible scenario. The problems will be addressed more in the Discussion section.
As seen in Figure 10A, the likelihoods are highest when the slip is low and the spreading rate is relatively high. The likelihoods also have a less significant high when half spreading rate is equal to slip. Slip rates are expected to be two times the half spreading rate. However, the results are worst when the slip has a 5-40 mm/yr difference from the half spreading rate. In Figure 10B, the data contain fewer patterns for $\alpha$ and thermal diffusivity. Likelihoods increase as $\alpha$ decreases. It might also seem that likelihood increases as thermal diffusivity increases with a low $\alpha$, but $\alpha$ represents the amount of effect temperature has on Youngs modulus in a given rock and at low $\alpha$ thermal diffusivity has no effect on the model.

**Full Dataset**

For the full dike dataset run from the starting parameters of 81 mm/yr half spreading rate, 7 mm/yr slip rate (right-lateral slip), $\alpha = 0.72$, and 57.8 thermal diffusivity, I have obtained final parameters and the final predicted dike poles. The full dataset ran for 365 evaluations which lasted for around 10 days. The run of the full data ended with the parameters with the highest likelihood of 101.2 mm/yr half spreading rate, 10.3 mm/yr slip rate (right-lateral), 0.52 alpha, and 68.6 thermal diffusivity. These results will be discussed more in the Discussion section.

The predicted and actual dike poles are shown in Figure 11. All of the predictions have a dike pole close to horizontal, while in reality, dike poles deviate around 30° from horizontal. The model predicts few dike poles around the azimuth of 045°, which matches the field data, and it seems as if the model predicts a similar rotation in data where dikes close to the ridge strike North South and dikes further from the ridge strike closer to East West.
Figure 10. Log likelihoods of the 23 data point subset of dike directions. The log likelihoods have been contoured to better show patterns, the darker the color, the more likely the parameters. (A) shows the half spreading rate versus the slip rate. Slip rates greater than half spreading rates were disallowed by the program due to an error in the implementation of the spreading rate. However, after subsequent analysis, the points with the highest likelihood are still those with low slip rate and high half spreading rate. (B) shows alpha versus the thermal diffusivity. Alpha was restricted between 0 and 1 and thermal diffusivity was restricted to positive values.
DISCUSSION

The results of the analysis have interesting implications on tectonic development of a ridge transform intersection. However, there are numerous possible problems with the model and implementation, which could lead to misleading results. First, I discuss the results and their implications. Then, I discuss the possible setbacks and problems with the model that could affect the results of the analysis.

Results

The final results have surprising results that are further supported by the runs of the small dataset. The most surprising result is the large difference between the spreading rate and the slip rate along the fault, including favorable likelihoods with right-lateral slip. Most previous models of ridge transform intersections predict a slip that is very close to the spreading rate. As mentioned in the modeling background section Behn et al. (2002) found that transform faults act as zones of weakness and predict a coupling of around 5%, which would indicate a slip that is 95% of the spreading rate. However, the results predict a coupling of 95% on the transform fault. The results also indicate a full spreading rate of 200 mm/yr which is at the highest range of spreading rates for mid-ocean ridges. The temperature variables are also outside of their expected values. Thermal diffusivity should range between 10 – 20 for basalts and not as high as 60. However, when looking at the hundredth dataset, it appears that the temperature variables do not have as much of an effect on the likelihood as the spreading rate and slip parameters. Overall, these results are very different from what I would expect based on past literature.

Additionally, comparison of the dike poles show that the model does not fit the data well (Fig. 11). Many of the problems with the model producing strange results could stem from the fact that the predictions are not very close to the observations in any of the models. I
Figure 11. Equal area lower hemisphere projection of the observed dike pole directions (A) and the predicted dike pole directions for the best fit model (B) colored by the easting with red as closer to the ridge and blue as further from the ridge. The strikes generally match while the dips are very different. (C) shows a histogram of the angle between the predicted dike pole and the observed dike pole with the density plotted in red and the average angle plotted with a dotted blue line at 49°. The angle is most clustered between 30°-50°.
will now investigate what problems could have occurred that would lead to different results from past literature and field data.

The model I am using has the advantage of being able to produce a velocity field and a stress field, but how well does this stress field match previous models? The Furlong et al. (2001) model does not investigate the stress structure of a ridge-transform intersection, so I must consider how the model’s stress outputs compare to other past models. Figure 12 shows the stress outputs of the final model compared to those of Phipps Morgan and Parmentier (1984). The stresses are especially different far from the transform (where they are not parallel to the ridge) and far from the ridge (where the stresses are almost perpendicular to what has been modeled in the past). The stress field is not as close to Phipps Morgan and Parmentier as the traction model, which puts the validity of the model into question. The model does not do as much to attempt to model the more realistic interactions of a ridge-transform intersection.

**Shortcomings of the Model**

In this section, I will discuss the possible reasons the results of this analysis differ from both previous models and the data collected in Cyprus. I will investigate problems with my implementation of the model, past studies to see if their models included measurements with more coupling, problems related to the use of PyLith in the model, and the differences in the stress fields.

**Problems with Implementation and Assumptions**

One problem with the implementation of the data is that I do not have a concrete way to determine the depth in the midocean ridge that the dikes originally formed. I can only approximate the depth of the dikes using the rock type that the dikes have intruded into. This approximation could cause temperature to have a misleading effect because the depth I
Figure 12. Plotted is the maximum horizontal compressive stress directions from the Traction model (A) compared to the maximum horizontal compressive stress direction based on previous Phipps Morgan and Parmentier (1984) results (B). The scale of the models is slightly different with a 60 km fault in (A) and a 100 km fault in (B). The stresses are less similar than the Phipps Morgan and Parmentier model. Near the ridge and transform the stress directions are similar but far from the fault at the outside corner the stresses curve in the opposite direction predicted in Phipps Morgan and Parmentier (1984).
assign to the dikes could differ significantly from their actual depth of formation, which could also affect the results of the data in other ways.

The second problem with the implementation arises when comparing the predicted dike poles to the observed dike poles. Based on the predictions, the model almost always puts the least compressive stress in the horizontal plane. All of the boundary conditions for the Displacement model are applied in the horizontal plane, which leads to the least compressive stress remaining in the horizontal plane. Dikes are predicted to form perpendicular to the least compressive stress, which leads to the Displacement model primarily predicting vertical dikes. The boundary conditions also lead to the majority of velocities lacking uplift components, which means that the dikes do not rotate from vertical based on the strain in the model. All in all, the model is not accounting for the 3D nature of the deformation compared to what is necessary to properly model the data. However, there has been recent discussion on the standard assumptions of working with dikes as a paleostress indicators. The model I am using assumes that dikes always form perpendicular to the least compressive stress direction. However, this assumes that dikes always form in tensile fractures. It has been proposed that dikes could also use shear fractures to form (Borradaile et al., 2010). This would indicate that dikes could form between 30 and 45 degrees from what our data predicts, which would fit better to the observed data (Fig. 11). In essence, the stress predictions could be correct while the data could be a bad paleostress indicator.

**Did Past Studies Test for High Coupling Rates?**

One possible reason the results are so different than past models is that no other models have tried to use higher coupling percentages. The highest coupling rate tested by Behn et al. (2002) is 15%, which is much lower than the results of this analysis. However, while Behn at all did not consider higher coupling rates, their reasons are geologically sound.
They argue that, based on Boettcher and Jordan (2001), seismic events can only account for 10-15% of the observed slip along oceanic transform faults, implying that 85-90% must be aseismic creep or off-fault deformation. Behn et al. (2002) interpret the analysis of Boettcher and Jordan (2001) as evidence for the transform fault being a zone of extreme weakness characterized by very little coupling and slow steady movement with the spreading rate. The results from the model can still account for the research done by Boettcher and Jordan (2001) if most of the motion on the oceanic faults is off-fault deformation. It is likely that the coupling is so high for the model because it needs to rotate the dikes a significant amount from their initial predicted orientations.

**Problems Related to PyLith**

PyLith is a software designed to model the stresses and deformations of short-term events such as earthquakes. It was not designed to be used on long-term deformation that occurs over millions of years. Because of the design of PyLith, and the length of computing time, I run PyLith as an instantaneous run to get the instantaneous stress field and velocity field and then use Python and C++ to compute what strain would occur for each specific dike over time using the velocities and stresses. However, in order to run PyLith instantaneously, I model the crust as an elastic material because viscoelastic and viscous materials cannot run instantaneously for velocity. As discussed earlier, using an elastic material forces the outputs to be displacements instead of velocities. I can convert these displacements to velocities, but only if I know the amount of time over which the displacement occurred, which can be determined by using displacement boundary conditions that are related to the spreading rate. The best option would be to expand on the Traction model in a way that can convert the displacements to velocities or find a program or method that can produce instantaneous
velocities, which PyLith cannot do and is possibly impossible based on continuum mechanics. The way the model is designed also removes a lot of the complexities of a true 3D system by applying the displacements to the whole side, which is true of a majority of past models.

A significant number of the past studies used 2D models (Fujita and Sleep, 1978; Phipps Morgan and Parmentier, 1984; Grindlay and Fox, 1993; Neves et al., 2004) or 3D models with boundary conditions applied over an entire face (Furlong et al., 2001; Behn et al., 2002). These previous models of ridge-transform intersections would have likely predicted entirely vertical dikes while the real data has a variable dip.

CONCLUSION

The goal of this analysis is to produce an inverse model that can follow the rigorous numerical forward modeling of past work on ridge-transform intersections and can then be compared to real data to find the best parameters for the data collected. The design of this model is of more significance than the results it produces. A combination of inverse and forward modeling can take into account real physics and dynamics, while also being related to real data collected in the field. This method has promise of working. However, due to a program that is not designed to work the way I am using it and problems with the implementation, the model was not very successful at modeling the data from Cyprus.

The model would be improved if it used a program that was better designed to run instantaneous velocities and stresses. It would also be valuable to use a model that was better able to replicate the stresses from past work while also providing meaningful velocities, which was not possible using PyLith and the Displacement model. However, even if the model was designed to run instantaneous velocities and stresses and replicated the stresses
from past work, it is likely that the predictions would have still been significantly different
from the dike pole data from Cyprus.

ACKNOWLEDGEMENTS

I am very grateful to my advisors Sarah Titus and Josh Davis for their support and
advice this project. I want to thank Sam Bacon for providing the foundational code for this
project and his help with learning PyLith. I would also like to thank the rest of the Geology
faculty for teaching me the geology I know and instilling in me a love for geology. I would
also like to thank Natalie Hummel for the help with editing and the rest of the geology
majors for the moral support during the process of working on this project. Finally, I want to
thank my family and friends for supporting me through my comps and helping me through
my time at Carleton. This material is based upon work supported by the National Science
Foundation under NSF-EAR 1151851 (Titus).
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