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NUMS 2021

The Northfield Undergraduate Mathematics Symposium (NUMS) is a joint event held with St. Olaf every year showcasing student research in mathematics and statistics. NUMS 2021 will be held virtually on **Tuesday, October 26, 2021** from **3:30pm-5:30pm**. The speakers from Carleton are Bryan Boehnke, Bowen Li, Shuhang Xue, and Yucheng Yang, and the speakers from St. Olaf are Ben Homan, Ella Koenig, and Luke Malek. A schedule of talks, as well as the Zoom link, will be emailed through the mast-interest listserve. Please come support your peers and hear about their research!

Putnam Signup Time is Here!

The William Lowell Putnam Mathematical Competition is a challenging exam focusing on mathematical insight and ingenuity. Typically, several thousand undergraduates across the United States and Canada participate, and the median score is usually less than 10 out of a possible 120. So if you get one of the twelve problems right, you're doing great! Whether you've participated in the contest before or are considering taking it for the first time, you'll probably enjoy getting experience with past Putnam problems (and learning some new problem-solving strategies) at our weekly problem-solving group, which meets every week on Wednesday, from 4:00 to 5:00 in CMC 328.

This year the Putnam will be held on Saturday, December 4. That's during our winter break, but we'll gladly make arrangements for you to take the Putnam at another college or university. If you'd like to sign up, or just have questions about the contest, contact Rafe Jones at rfjones@carleton.edu. There's still time to sign up, but do so as soon as possible, and no later than November 1.

What's the Math and Stats Department Teaching Next Term?

Have you checked your registration time yet? Made a list of classes you're hoping to take next year? Let the course descriptions below guide you into an adventurous new term with the Carleton Department of

Mathematics and Statistics! There's something for everybody, from topology to applied regression. Find out more below.

Math Classes

Math 206: A Tour of Mathematics
Instructors: Many of us
Time: Fridays only, 6a (3:30-4:30)
Prerequisites: None

Are you considering a math major? Then you should definitely consider taking this series of eight lectures by math and stats faculty. They will present some striking ideas, concepts and results in an attempt to convey the breadth, beauty, and power of their areas. Credit for this 1-credit course will be based on attendance (and, at times, participation) only. Math 206 is offered annually, and you are allowed to register for it twice, in consecutive years: There should be essentially no overlap with the 2021 version of the Tour. Contact Mark Krusemeyer if you have logistical questions.

Math 236: Mathematical Structures
Instructor: MurphyKate Montee
Time: 3a
Prerequisites: Math 232, and Math 120 or Math 211, or instructor permission

How do we know that mathematical theorems are true? You may have seen examples of mathematical proofs in previous classes, but how do you read and understand one? How do you write one yourself? The primary goal of this class is to teach you how to read and write proofs; in other words, you'll be learning how to communicate mathematical ideas like a mathematician. As such, you should expect to do a lot of writing, and probably less calculating than you are used to in a math class! If you've ever wondered why some math calculation works, this is the place to get the answer.

Along the way, we'll cover concepts including set theory, formal logic, different "sizes" of infinity, and even dig into old favorites like functions. We'll look at some statements that we've taken for granted in previous classes (e.g. the square root of 2 is irrational, there are infinitely many prime numbers, we can factor any natural number into primes,....) and work out how to write a convincing proof of them. Plus, Structures is a prerequisite for most of the upper-level math courses!

Math 240: Probability
Time: 4a
Instructor: Laura Chihara
Prerequisite: Math 120 or 211

If the "immortal monkey" randomly strikes keys on a keyboard for eternity, what is the probability that it will eventually produce the complete works of Shakespeare? If in a small town, out of 12 accidents that occurred in June 1986, four of them occurred on Friday the 13th, would this confirm your hunch that "13" is unlucky?

Probability is a fundamental branch of mathematics and is the foundation for all methods of statistical inference. In this course we will use the tools of counting and calculus to model random events, compute probabilities, and have lots of fun with balls in urns, poker hands, and coins and dice (fair or otherwise).

Math 241: Ordinary Differential Equations
Instructor: Kate Meyer
Times: 2a and 3a
Prerequisites: Math 232 or instructor permission

Differential equations are a fundamental language used by mathematicians, scientists and engineers to understand and describe processes involving continuous change. In this course, we will study differential equations from both a practical and theoretical point of view. Our focus will be on developing differential equation models and exploring the mathematical ideas that arise within these models. Examples may include Earth's energy balance, mechanical vibrations, love affairs, competition and cooperation of species, and more! The science will stay at an elementary level; our focus will be the mathematical ideas that arise in these models.

Math 331: Real Analysis 2
Instructor: Rafe Jones
Time: 2a
Prerequisite: Math 321 or instructor permission

How can we talk about the size of a set of real numbers? How can we integrate as many real-valued functions as possible? The answers to these questions turn out to be closely related, and take us beyond notions of size from Mathematical Structures (countable vs uncountable) and notions of integration from calculus (the Riemann integral). By the end of the course, we will be able to "measure" a vast array of subsets of the real numbers, and integrate functions that leave the Riemann integral gasping for breath. We will also touch on how this notion of measure forms the underpinnings for a rigorous formulation of probability theory.

The course covers Lebesgue measure, the Lebesgue integral, an introduction to general measure theory, and Banach and Hilbert spaces. Not only is this your chance to "integrate" your knowledge of functions, it is also an opportunity to better your understanding of the legal interchange of limit operations. Rumor has it that the Cantor set may also make an appearance. The flavor of the course will be similar to a graduate-level course in analysis. If there is a possibility that you are headed for graduate school in mathematics or a related field, this course comes highly recommended.

Math 342: Abstract Algebra
Instructor: Claudio Gómez-Gonzáles
Time: 4a
Prerequisites: Mathematics 236 or instructor permission

Algebra is a pillar of modern mathematics concerned, broadly, with the study of structure. As such, algebraic problems arise across mathematics, the physical sciences, art, and everyday labor: from particle physics to quantum chemistry to textile production. In this course, we will draw inspiration from one of the oldest known mathematical problems—solving polynomials—and trace through some of the historic struggles that have informed modern algebraic theory. These considerations will take us through fields, rings, and groups, which generalize ordinary arithmetic systems like numbers, matrices, permutations, "clock math," and the like. Along the way we'll see how a few simple axioms can give rise to a remarkably rich theory and a zoo of fascinating structures.

Math 349: Methods of Teaching Mathematics
Instructor: Deanna Haunsperger
Time: 4-5c (Tuesdays and Thursdays 1:15-3:00)
Prerequisite: Junior or Senior Standing and permission of the instructor

How is mathematics taught? You've certainly seen mathematics taught, and if you're a tutor or have a younger sibling, you've probably done some teaching. Is there a best way to teach? How do students learn mathematics? How do teachers plan out their days? What is important when you're in front of the class? Through readings and observations and practice, we'll discuss these questions and you'll develop your own answers. Enrollment in this course requires a time commitment outside of class observing in the Northfield public schools.

Math 354: Topology
Instructor: Steve Scheirer
Time: 5a
Prerequisites: Mathematics 236 or instructor permission

At its heart, topology can be viewed as the abstract study of shape. We consider two "shapes" to be "the same" if we can deform one shape into the other. This deformation can involve things like stretching, shrinking, crumpling, and so on, but we can't tear anything or fill in any holes. You can probably convince yourself that you can deform a square into a circle in this manner, so from a topologist's point of view, a square is indistinguishable from a circle. Likewise, a topologist can't tell the difference between a football and a basketball or indeed the difference between a coffee mug and a donut. Instead, we look for properties which are common between spaces which are "the same," or better yet, properties that would allow a topologist to tell the difference between two spaces. Can a topologist tell the difference between a coffee mug and a football? We'll know the answer by the end of the term!

Math 395: Algebraic Geometry Seminar
Instructor: Mark Krusemeyer
Time: 2-3c (Tuesdays and Thursdays 10:10-11:55)
Prerequisites: Math 342 or permission of the instructor

Algebraic geometry, which can be thought of as starting from a blend of abstract algebra and analytic

geometry, is one of the most highly developed areas of mathematics. Long considered a "pure" subject, it eventually found computational applications in areas such as computer-aided design and robotics. The techniques of modern algebraic geometry are considered to be among the most powerful mathematical methods known; for example, algebraic geometry was involved in the Wiles-Taylor proof of "Fermat's Last Theorem," even though, on the surface, the statement of that theorem involves neither abstract algebra nor geometry.

Although Math 395 is a "senior seminar," non-seniors are welcome to take it. The seminar provides a rare opportunity to get a head start on algebraic geometry (which is typically studied at the graduate level). While the precise topics covered will depend on the participants' background and interests, one entry point is likely to be the study of plane algebraic curves, that is, curves given by polynomial equations P(x, y) = 0. We'll see how geometric properties of these curves, such as the directions in which they go off to infinity or the existence and location of "singular points" where there is no unique tangent line, can be described and predicted by looking at the polynomial P(x, y) and at a certain ring constructed from that polynomial. We'll also see how to generalize some of this material to more complicated geometric objects in higher dimensions, and probably study more abstract "geometric" settings that arise from general commutative rings.

A fair amount of the material in this seminar will be presented by the participants (just how much depends, for one thing, on how many of you sign up); there will be no exams, and grades will be based on class presentations, homework assignments, and general class participation.

Stats Classes

Stat 220: Introduction to Data Science
Instructor: Deepak Bastola
Time: 4a
Prerequisites: Stat 120, Stat 230 or Stat 250

This course will cover the computational side of data analysis beyond exploratory data analysis and regression modeling. We will begin with extracting useful information from unstructured and messy data. We will use extensive visualization tools like ggplot and interactive platforms called Shiny to visualize our data. We will learn some of the modern classification, predictive models and other computationally-intensive statistical learning methods. We will learn the basic tools to help us effectively communicate statistical results so that we can tell interesting stories of data with pretty visuals and sound statistical reasonings. We will primarily use the stat software R in this course.

Stat 230: Applied Regression Analysis
Instructor: Andy Poppick
Time: 5a
Prerequisites: Stat 120 or Stat 250

How is air pollution associated with mortality? How is flooding associated with snowmelt? Answering these kinds of questions require statistical models that describe the distribution of a response variable in terms of predictors. In this course we'll cover regression models for continuous response variables

(ordinary linear regression) as well as models for binary and count data (logistic and Poisson regression). Crucial practical questions to be addressed include: How do I build a regression model? Is my model a good description of my data? What do I mean by "good"? What do I want to use my model for? Can I answer substantive questions with my data and model, and can I accurately quantify my uncertainty? Can I make predictions with accurate uncertainties? Can I build a better model? The emphasis will be on data analysis and communication of results in realistic settings. We will make frequent use of R.

Stat 250: Introduction to Statistical Inference
Instructor: Adam Loy
Time: 3a
Prerequisites: Math 240

In probability we assumed perfect knowledge of a population, and used this knowledge to grapple with properties of random samples. In statistics, we flip the script: we now observe a random sample and wish to learn about our population. For example, we will discuss how to estimate the parameters for probability distributions based on data, rather than assuming they are known.

This course uses probabilistic and computational tools to introduce statistical inference. While we will discuss the theory underlying these inferential methods, we will balance this theory with data-driven applications that illustrate how these methods are applied. Modern statistical practice also requires computation, so this class will introduce you to the R environment for statistical computing.

Stat 260: Intro to Sampling Techniques
Instructor: Katie St. Clair
Time: 2a
Prerequisites: Stat 120 or Stat 250

This course covers a wide range of statistical sampling techniques that are used to make inferences about a population. We will discuss how to form estimates and quantify the sampling error using "sampling weights" when data is collected using sampling designs that are more complex than a simple random sample. We will also cover strategies for determining an "optimal" sampling design when resources (time/money) are limited. Time permitting, we may also cover how sampling weights are used in data visualization, regression models or chi-square hypothesis tests. Applications will be drawn from both the natural and social sciences, and we will use the R survey package extensively throughout the course.

Stat 285: Statistical Consulting
Instructor: Andy Poppick
Time: Tuesdays 10:10-11:55am
Prerequisites: Statistics 230 (formerly Mathematics 245) and instructor permission

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

Upcoming Events

Week 7

Tuesday October 26, 3:30-5:30pm
NUMS — Zoom

Job, Internship, & Other Opportunities

Predoctoral Research Fellows — Stanford Institute for Economic Policy Research

The SIEPR/Economics Predoctoral Research Fellows program is a full-time, one- to two-year post-baccalaureate program designed to prepare individuals wishing to gain valuable training and experience toward a career in academic research in economics or public policy. For priority consideration, candidates should submit application materials by Friday, November 12. Positions will remain open until filled.

Apply at siepr.stanford.edu/programs/predoctoral-research-fellowship-opportunities.

Director's Financial Analyst — Consumer Financial Protection Bureau

This unique, two-year rotational fellowship sits at the intersection of the federal government and the financial services industry. Director's Financial Analysts are given the opportunity to hone analytical and problem-solving skills while helping to make consumer financial markets work for Americans.

All analysts will complete developmental rotations in offices throughout the CFPB. These rotations are designed to provide exposure to the analysis, strategy, research, education, policy development, supervision, enforcement, and rulemaking activities throughout the Bureau. In a short period of time, analysts will play an integral role in everything the Bureau does, from rigorous data-driven policy creation and market monitoring to supervision of market participants.

The DFA program is recruiting for full-time positions that begin in June 2022. Recent graduates who will have received an undergraduate degree on or after April 2019 and before June 2022 are eligible to apply. Those interested should apply at www.consumerfinance.gov/careers during our live application window on USAJobs. The application will be open from December 6 through January 3.

To be notified when the application becomes available, students and alumni can send an email expressing interest to CFPB_DFA_Program@cfpb.gov.

Carleton College Investment Office Intern (Summer 2022)

The Carleton Investment Office, located in Edina, MN is seeking a full-time intern for summer, 2022. Carleton College's Investment Office is responsible for the ongoing management of the endowment portfolio. Spending drawn from the endowment each year funds a meaningful portion of the College's annual operating budget, supporting financial aid for students, academic programs, and faculty and staff salaries.

The intern will have an opportunity to learn about the endowment, how it works, and how it is managed.

Interns will learn the process of asset allocation and manager selection, and will gain exposure to a wide range of investment asset classes including public equity, private equity, hedge funds, fixed income, and real assets. Interns will participate in meetings and conference calls with fund managers, conduct analysis on the portfolio, and assist with administrative and operational tasks.

Apply at carleton.joinhandshake.com/jobs/5505639.

Kelsey, Jeff Dsida '14 (Econ), and Zong Lim '21 (Econ) will be on-campus on Nov. 11th to promote the Carleton Investment Office and to recruit for the internship. Find more information and how to attend at apps.carleton.edu/career/about/events/?event_id=1000149832&date=2021-11-11.

Junior Trader — Diffuse

Diffuse is a hedge fund incubator and currently offering a part-time job working for a high growth crypto asset management team. They offer a market rate salary and always convert talented students to a full-time role upon graduation with full benefits, a generous bonus, and equity in Diffuse. Find thge full description and apply at diffusefunds.com/career/jr-trader-position-part-time/.

Problems of the Fortnight

To be acknowledged in the next *Gazette*, solutions to the problems below should reach me by noon on Tuesday, November 2.

1. A prime number p is called a *Germain prime* (after the number theorist Sophie Germain) if $2p+1$ is also a prime (which might or might not be another Germain prime). For example, 1009 is *not* a Germain prime, because $2 \cdot 1009 + 1 = 2019 = 3 \cdot 673$ is not prime. On the other hand, 509, which is prime, is a Germain prime, because $2 \cdot 509 + 1 = 1019$ is another prime; in fact, $2 \cdot 1019 + 1 = 2039$ and $2 \cdot 2039 + 1 = 4079$ are also prime, so $509, 1019, 2039$ are all Germain primes. Alas, $2 \cdot 4079 + 1 = 8159$ factors as $41 \cdot 199$, so our sequence of "linked" Germain primes ends with 2039. Now for the problem: Does there exist an infinite sequence of "linked" Germain primes, such that all numbers in the sequence are prime and each number q in the sequence is followed by $2q + 1$? Why, or why not?

2. a) Find, with proof, all (real-valued) functions f with the following properties:
       $f(x)$ is defined for all real x; f is continuous at $x = 1$; $f(x^2) = xf(x)$ for all real x.
    b) If we drop the condition that f is continuous at $x = 1$, will there be additional functions that have the other properties?

Sebastian Vander Ploeg Fallon solved the second problem posed October 8, and should consult with Sue Jandro about picking up an item from the B.B.O.P. Although I have reason to believe that a student solution to the first problem may be on its way (and I hope so), as of press time it hasn't arrived. Meanwhile, correct solutions to both problems from October 8 were submitted by "Auplume". Enjoy the new problems!

- Mark Krusemeyer