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Jigsaw puzzles and broken bones

This comps is about putting things back together. The things might be cracked eggs, broken bones, shattered vases, or even jigsaw puzzles. There are two big mathematical and computational tasks underlying the process of putting things back together:

1. deciding when and how two pieces of a broken object fit together, and
2. combining all of these pairwise fits into a fully assembled object.

Task 1 can be approached by using *shape invariants* – quantities computed from the object which encode shape in some way that is independent from the way we view or represent it – and comparing these shape invariants to search for ways that pieces can fit. Task 2 involves some kind of optimization; among all possible ways of assembling the object we need to find a *best* one.

We'll choose one of the following topics based on the interest of the group:

Topic 1: Fast and accurate jigsaw puzzle assembly. Carleton students have succeeded in automatic assembly of jigsaws with 104 pieces, but the process gets slow very quickly as the number of puzzle pieces increases. We'll build on this previous work to create more robust and faster assembly strategy, and compare the effectiveness of different shape invariants for detecting when a pair of jigsaw puzzle pieces fit together. We can also investigate how assembly strategies need to change for non-rectangular jigsaws (e.g. those with points at which 3 or 5 or more pieces meet). As a final test of our work, we could pit human against computer in a jigsaw puzzle solving duel.

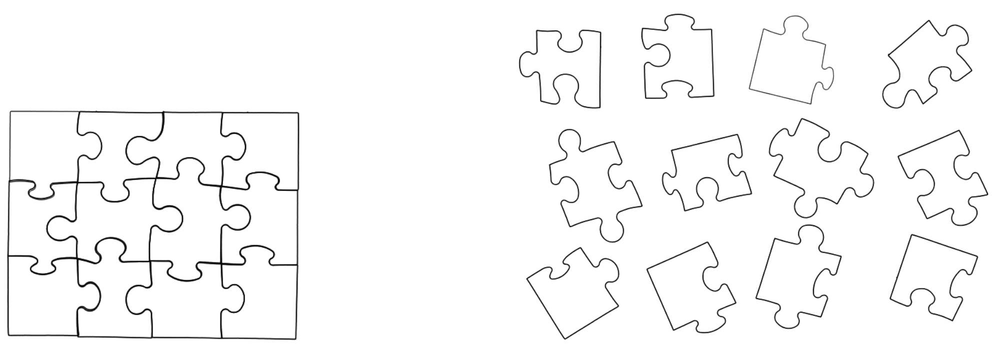

Figure 1: Algorithmic assembly of a digital jigsaw puzzle.

Topic 2: Cracked eggs and 3d jigsaws. A cracked egg is messy, and so is the problem of automatically reassembling its shell. Previous work with students has succeeded in assembling an ostrich egg broken into 15 pieces using 3d scan data from the egg; and a similar method works to assemble synthetic 3d jigsaw puzzle data. This assembly has been limited to small synthetic puzzles with fairly simple geometry and a single egg's worth of data. We'll break and scan some eggs, wrestle with the messy data that results, and refine the approach to 3d shape matching when shapes might be jagged or vary widely in size. We can also create more geometrically complex synthetic 3d puzzles, test assembly methods, and try 3d printing them to see if they'll fit together in real life.

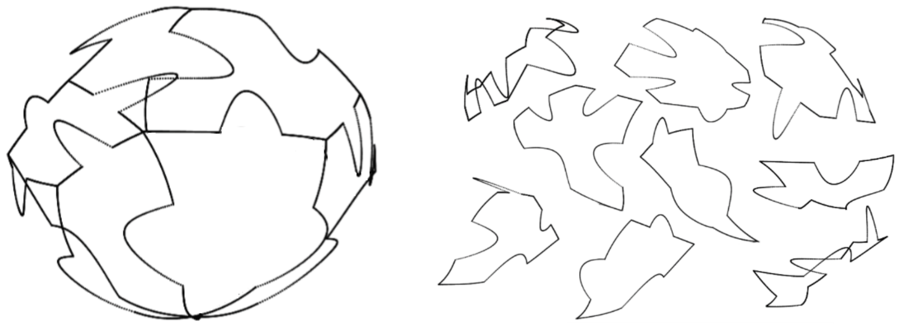

Figure 2: Algorithmic assembly of a 3d digital jigsaw puzzle.

Topic 3: Broken bones and surface matching. A group of mathematicians at the University of Minnesota recently began studying a collection of digitized broken bones. These bones come from elk, and have two kinds of breaks: those made by predators, and those made by tools. The group is interested in two questions: first, can shape invariants or other geometric information help classify the type of break? Second, can these bone pieces be reassembled? These questions have driven the recent development of new shape invariants based on surface geometry (the bone break is a jagged surface, rather than a curve or edge as in the case of eggs and jigsaw puzzles), but very little has been done with the reassembly process. We'll work with this research group to learn about some of these new shape invariants, develop methods for computing and comparing them, and test these comparison methods to try to reassemble synthetic broken surfaces and real broken bones.

Prerequisites: There are no specific prerequisites. I recommend having taken at least one CS course or equivalent computing experience.