Directed Reading in Geometric Group Theory
Winter/Spring Comps
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Geometric Group Theory (GGT) is the study of groups using techniques of geometry. The key insight of GGT is that one can prove properties about groups by considering spaces that the group acts on. One corollary of this is that you can draw a 'picture' of a group in a mathematically rigorous way. The standard example (but not the only example!) of this is the *Cayley graph* of a group G. This depends on a *generating set* S for the group, i.e. a set S so that every element of the group can be written as a product of elements in S. The Cayley graph $\mathcal{C}(G, S)$ is a directed, labelled graph with the following vertices and edges:

- The vertex set of $\mathcal{C}$ is the elements of G.
- A directed edge (g, h) connects two vertices g, h if and only if $h = gs$ for some $s \in S$. This edge is labelled s.

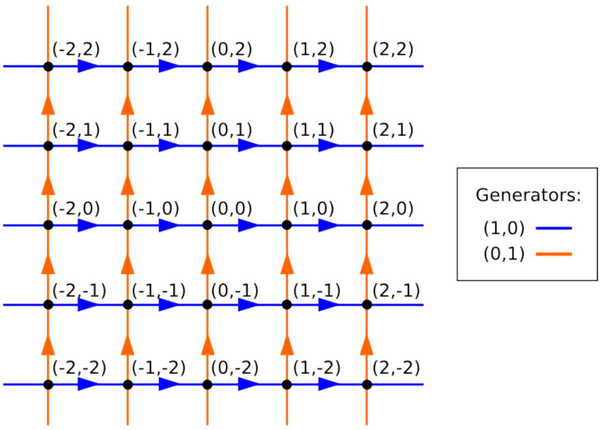

Figure 1: The group $\mathbb{Z}^2$ can be generated by $(0,1)$ and $(1,0)$. The corresponding Cayley graph is shown here.

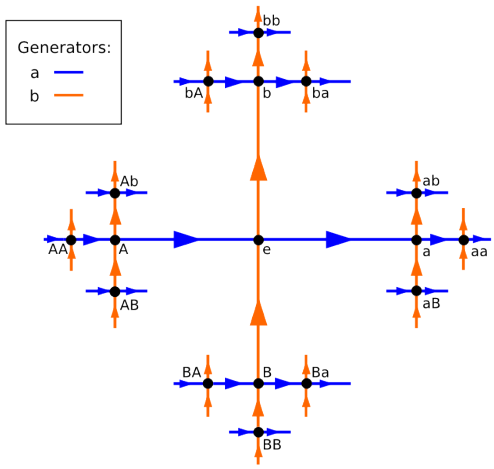

Figure 2: The free group F_2 on two elements is generated by two elements a and b. The corresponding Cayley graph is shown here.

At first, it seems like this graph would depend heavily on the choice of generating set S. But actually, there are strong geometric similarities between any two Cayley graphs that represent that same group!

In this comps project, we'll begin learning about generators and relations of groups, and what a free group is. We'll prove that every subgroup of a free group is free, and then show that acting (sufficiently nicely) on a tree is equivalent to being a free group. There will be room to choose what else we look at after that point. Possibilities include solving the *word problem* in *hyperbolic groups*, exploring different families of groups (such as *Right-angled Artin groups, Coxeter groups, surface groups*), or learning about *finite state automata*. Our primary source will be *Office Hours with a Geometric Group Theorist*. We will supplement that source with other papers.

Suggested (but *not* required!) prior knowledge: Familiarity with groups, like you would have from Math 342 (Abstract Algebra).