Goodsell Gazette

Carleton College — February 14, 2020

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Comps Talks

Come support your classmates and friends at their comps talks next week! Group comps talks will take place 3:30-6:30pm in Boliou 104 on both Tuesday, February 18 and Thursday, February 20. Take a look at what they'll be speaking about below, then be sure to stop by and support them while they show what they've learned; you're likely to learn a thing or two as well!

Tuesday, February 18 — Boliou 104

Title: Respondent-Driven Sampling
Speakers: Katie Chavez, Bryan Kim, Jay Na, Aaron Prentice, Jin Ruan, J. Liralyn Smith
Time: 3:30pm

Abstract: Respondent-driven sampling (RDS) is a "snowball" type of sampling method primarily utilized for reaching hidden, stigmatized populations whose sampling frame is unknown. RDS consists of several waves of incentivized peer-to-peer recruitment that finishes when the desired sample size is met. With this sampling method there is a tendency to oversample highly connected people who share similar characteristics which induces homophily in the data. In this talk, we will discuss how Spiller (2009) specifically focused on the role of homophily in regression modeling of RDS data and our attempt to replicate his findings. We will also discuss a statistical simulation to analyze the estimation performance of a basic logistic regression model with RDS data. The estimator properties of a random effects model with RDS data will be examined as well.

Title: Pathologies of the Axiom of Choice
Speakers: Todd Johnson, Adam Kral, Aaron Li, Justin Soll
Time: 4:30pm

Abstract: We will examine the following questions. Cauchy's Functional Equation is defined as follows: $f(x+y) = f(x)+f(y)$ for all x, y. Does every solution of this take the form $f(x) = kx$ for some k? Is R^2 group isomorphic to R? How many different sets can we get by applying complement, closure, and the "D operator" to some subset of R? To answer these disparate questions in algebra and analysis, we will need to explore how some abstract assumptions from set theory, like the Axiom of Choice, can have a direct impact. The answers turn out to share some surprising properties.

Title: (K)not: A Presentation — An exploration of ribbon knots in the plane
Speakers: Mack Journell, Weijia Ma, Alex Rafkin, Paul Reich
Time: 5:30pm

Abstract: How long does your shoelace need to be for you to tie it? Have you ever studied a DNA double helix? We are interested in these examples of physical knots that appear frequently both in daily life and in the natural sciences. We study folded ribbon knots, which are 2-dimensional knots embedded in a plane, and their ribbonlength, which is defined as the minimal length-to-width ratio of all possible representations for the knot. In our talk, we use a variety of grid frameworks to visualize folded ribbon knots, find a linear upper bound for rational knots, and attempt to find an upper bound for all knots by studying braids.

Thursday, February 20 — Boliou 104

Title: Fermat's Last Theorem: Kummer's Special Case
Speakers: Sam Chen, Marietta Geist, Alex Kiral, and Brody Lynch
Time: 3:30pm

Abstract: Prior to its resolution by Wiles and Taylor in 1994, Fermat's Last Theorem had plagued mathematicians for 350 years. In 1847, Gabriel Lame provided an incorrect proof of the problem which used complex roots of unity to reformulate the problem. However, his mistake motivated the study of unique factorization in terms of algebraic structures, which eventually led to the founding of algebraic number theory. In this talk, we explore these algebraic structures and outline a proof of Fermat's Last Theorem in a special case due to Kummer.

Title: The Story of the Alternating Sign Matrix Conjecture
Speakers: Ernest Matthew Finney-Jordet, Dean S. Gladish, Ben Schwartz
Time: 4:30pm

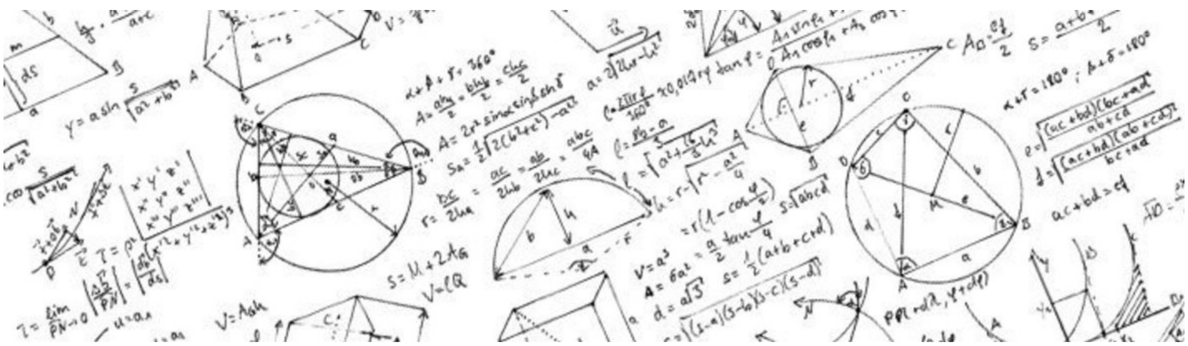
Abstract: An alternating sign matrix (ASM) is a square matrix made up of 0s, 1s, and -1s where each row and column must sum to 1 and the non-zero entries in each row and column must alternate in sign. In the early 1980s, William Mills, David Robbins, and Howard Rumsey discovered alternating sign matrices and made a conjecture about their quantity. How could we count these matrices? We will journey through the proof of the ASM conjecture using a set of objects called plane partitions, specifically totally symmetric self complementary plane partitions. In the end, we will discover that there exists a bijection between ASMs and these TSSCPP's, although a direct proof remains elusive.

Title: Optimizing Manufacturing at 3M
Speakers: Joseph Caradimitropoulo, Luanqi Chen, William Thompson, Yangqiaoyu Zhou
Time: 5:30pm

Abstract: How can we use mathematics to make a manufacturing process more efficient? We spent two terms consulting on this question with employees at 3M, a company you probably know for products such as Command Strips, Scotch tape, and Post-it notes. In this talk, we will outline the manufacturing problem our industry partners in the 3M Films Division posed for us, and the methods we used to approach it. We will explain the factors that influence a manufacturing schedule and how we translated these factors into mathematical language. By taking into account cost constraints and fluctuations in demand, we embarked on a computational modeling process of balancing efficiency with expense, which resulted in a mathematical model and a Python simulation that we hope 3M will build off of and use. Come and learn about 3M and the work we did with them!

What's the Math and Stats Department Teaching Next Term?

Have you checked your registration number yet? Made a list of classes you're hoping to take next year? Let the course descriptions below guide you into an adventurous new term with the Carleton Department of Mathematics and Statistics! There's something for everybody, from number theory to applied regression â€” find out more below.



Math 236: Introduction to Mathematical Structures
Instructor: Deanna Haunsperger **Time**: 2a (Sophomore priority)
Instructor: Mark Krusemeyer **Time**: 3a
Prerequisite: Math 232 or permission of instructor

How do we prove mathematical statements? How do we even think of possible statements, and what makes us suspect that a particular statement may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with deep roots that tap into the wisdom of many generations. Still, if you've ever wondered "How did anyone come up with that?", or "How can you really be sure of that?", about some mathematical result, taking this course may help dispel some of the mystery. We'll explore various concepts, especially from set theory, that are indispensable for most areas of advanced mathematics, and we'll spend considerable time developing theorem-proving and problem-solving skills. Along the way we'll take a new and closer look at some old friends, such as functions and relations: What are they really? In the final part of the course we'll use functions to compare "sizes" of various infinite sets. For example, we'll see that despite appearances, there are not any "more" rational numbers than there are integers; on the other hand, there are "more" real numbers than rational numbers. If you're considering a math major, taking this course should help you decide; also, "Structures" is a prerequisite for the majority of upper-level math courses.

Math 241: Ordinary Differential Equations
Instructor: Kate Hake

Time: 3a
Prerequisite: Mathematics 232 or instructor permission

In this course, we will study equations which express a relationship between variables and their derivatives. These equations, called differential equations, arise in many contexts of mathematics and social and natural sciences. The course will cover topics such as solving differential equations; numerical, analytic and graphical solution methods; solutions and spaces of solutions; linear systems; linearization; qualitative analysis of both differential equations and linear systems of differential equations; and structures of solution spaces.

Math 244: Geometries
Instructor: Deanna Haunsperger
Time: 4a
Prerequisite: Math 236

Goethe described it as "the fountain of all truth," Plato said it's how god thinks, PoincarÃ© said it's how you think, Edna St. Vincent Millay called it, "Beauty bare." It has been inspiring poets, philosophers, scientists and schoolboys/girls for 3000 years. It's geometry. Come see what all the fuss is about. We'll start with a quick revisit to Euclid's Elements, quickly skim over a couple of millennia of progress and then wallow in the creations of the last few centuries. We'll learn some fabulous theorems about circles and triangles that, had they been known, would have delighted Euclid. Then we'll wander in non-Euclidean space and learn some stuff that would have absolutely flipped him out--it should have the same effect on you.

Math 245: Applied Regression Analysis
Instructor: Tom Madsen
Time: 5a
Prerequisite: Math 215 (or equivalent) or 275

Model-based thinking is at the core of statistics. Simple linear regression — modeling a response variable as a linear function of an explanatory variable— is only the beginning. This course presents more advanced techniques for regression modeling, including models with many explanatory variables (multiple regression) or a categorical response (logistic regression). We will apply these techniques to a wide variety of data sets, with an emphasis on model building and checking and statistical writing. We will use the statistical software R extensively.

Math 275: Introduction to Statistical Inference
Instructor: Katie St. Clair
Time: 2a
Prerequisite: Math 265

Statistics is the art and craft of studying data and understanding variability. Though mathematics (in particular, probability) governs the underlying theory, statistics is driven by applications to real problems. We will cover basic statistical inference as well as modern computational approaches, all in the context of investigating interesting questions that arise in scientific and public policy settings. We will use the software package R.

Math 280: Statistical Consulting
Instructor: Adam Loy
Time: Tuesday only, 2/3c
Prerequisite: Math 245 and instructor permission

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

Math 285: Introduction to Data Science
Instructor: Adam Loy
Time: 1a
Prerequisite: Math 215 or Math 275

This course will cover the computational side of statistics that is not typically taught in an intro or methodology focused course like regression modeling. Most of data you encountered in your first (or second, or third, ...) stats course were contained in small, tidy .csv files with rows denoting your cases and columns containing your variables. The only messiness to these data may have been some missing values (NAs). We will start this course in data science by learning how to extract information from data in its "natural" state, which is often unstructured, messy and complex. To do this, we will learn methods for manipulating and merging data in standard and non-standard formats, data with date, time, or geolocation variables, text processing and regular expressions, and scraping the web for data. To effectively communicate the information contained in these data, we will cover data visualization methods (or, as statisticians often call it, EDA) that go beyond a basic histograms or boxplots, including methods for creating interactive graphics. We may also cover some modern computationally-intensive statistical learning methods. We will primarily use the R programming language in this course.

Math 295: Seminar in Set Theory
Instructor: Gail Nelson
Time: 2a
Prerequisites: 236 or instructor permission

Is there a set of all sets? If you allow the collection of all sets to be considered a set, the result, as you encountered in Math 236, is Russell's paradox. So what exactly can be considered a set? We attempt to answer this by working with a set (pun intended!) of axioms. We'll begin by building familiar number systems (the natural numbers, the rationals and the reals) from these axioms. Then our ambitions get really grandiose, as we proceed to construct successively larger infinite numbers, and finally, the entire mathematical "universe." Along the way, we will encounter cardinal numbers, ordinal numbers, and transfinite induction. Since its inception, set theory has been a source of beautiful and, at times, disturbing results. Attempts to reduce all of mathematics to the language of sets have provided insight into unifying underlying structure. But set theory has also led to strange and counterintuitive â€œalternate realitiesâ€ for mathematics.

Math 312: Elementary Theory of Numbers
Instructor: Alex Barrios
Time: 4a
Prerequisite: Math 236

Interested in numbers? Want to unlock their hidden secrets and structure? Then this is the class for you! In this course, we will follow in the footsteps of Euler, Gauss, Germain, Dirichlet, and many others in their quest to solve Fermat's marginal notes! Following the passing of Pierre de Fermat in 1665, his son published forty-eight mathematical claims that his father had made (without proof!) in the margins of his copy of Diophantus' Arithmetica. These claims mystified mathematicians for centuries and as part of this course, we will see how their quest to solve these infamous marginal notes led to the discovery of important theorems as well as the foundations of modern cryptography! In particular, we will prove Euler's Theorem on primes which are the sum of two squares, Gauss's Law of Quadratic Reciprocity, and Germain's proof of the first case of Fermat's Last Theorem for exponent 5. This course only assumes MATH 236: Mathematical Structures and begins by a further exploration of Euclid's work in number theory culminating in a proof of the Fundamental Theorem of Arithmetic: Every positive integer factors uniquely into a product of primes.

Math 321: Real Analysis I
Instructor: Owen Biesel
Time: 5A
Prerequisites: Mathematics 236 or instructor permission

How much can we trust the rules of calculus? Up till now, we've mostly looked at examples where everything works out nicely, but if we look hard, we can find a convergent series whose sum depends on the order of the terms, a multivariable function whose mixed partial derivatives aren't equal, a function that doesn't equal the derivative of its integral, and more. So how can we ever apply calculus to real-life problems and trust the results? To answer, we'll go back to the beginning and put the concepts of "integral," "derivative," "continuity," "limit," and even "function" and "real number" on a solid theoretical footing. You'll practice using proofs to understand exactly when the principles of calculus hold, a skill with enormous importance for the theories of differential equations, dynamical systems, economics, and probability. This course is highly recommended for anyone considering grad school in math or statistics.

Math 341: Partial Differential Equations
Instructor: Rob Thompson
Time: 3a
Prerequisite: Math 241

About 200 years ago, Jean Baptiste Fourier studied the way that heat moves through a flat metal plate via a partial differential equation called the heat equation. Trying to describe his observations mathematically, he did a seemingly simple thing: he expressed the heat distribution as a sum of sines and cosines (a "Fourier series"). Expressing the complicated behavior of heat in terms of simpler functions gave Fourier powerful insight into the behavior of the heat equation. Fourier's idea revolutionized pure and applied mathematics. In this course, we'll learn the fundamentals of partial differential equations and make a tour of Fourier's revolution. We'll examine various interesting PDE (including the heat equation) and their applications to wave propagation, heat conduction, elastic equilibrium, and more. We'll also develop ideas from Fourier analysis as needed to access information about the solutions to the PDE we study. Feel free to contact me (rthompson) with any questions!

Math 345: Advanced Statistical Modeling
Instructor: Laura Chihara
Time: 3A
Prerequisites: Math 245, Math 275, and familiarity with matrix algebra.

Suppose we want to see what socioeconomic factors are important in explaining math test scores for fourth graders in several school districts. In each district, there are three schools and in each school, there are two classes of fourth graders. Now scores of students within the same class may be impacted by the effectiveness of their teacher, and the scores of students in the same school may be impacted by the influence of the principal or school philosophy. And finally, scores for students in the same district might be impacted by the resources (ex. property tax revenue). How can we model these test scores while taking into account the dependence within each of these clusters? In this course, we will investigate modeling in situations where the assumptions that we made in multiple regression models in Math 245 are violated. We will also learn general linear models of which logistic and Poisson models are special cases, as well as extension of these models.

Math 352: Topics in Abstract Algebra
Instructor: Rafe Jones
Time: 2a
Prerequisite: Math 342 or instructor permission

If you liked Abstract Algebra, then this course will give you further helpings of the same great ideas. Our topic this year is Galois theory, which is quite different from last year's Math 352 (and you can take this course for credit if you took Math 352 last year). Galois theory is roughly the study of â€œalgebraic conspiraciesâ€ among the roots of polynomials, and as with most conspiracies, they can be very difficult to detect. We will introduce a group — called the Galois group of the polynomial— that detects all such conspiracies and reflects them in its structure. More precisely, the Galois group is the automorphism group of a certain field extension associated to the polynomial, thus giving an unexpected, deep connection between fields and groups. This connection is used widely elsewhere in mathematics, especially within algebra. It allows for a spectacular resolution of a problem that plagued humanity for centuries: can one generalize the quadratic formula to polynomials of higher degrees? In the Renaissance mathematicians had found such formulas to solve cubic and quartic equations, but the mythic quintic formula eluded everyone. Something about degree 5 (and higher) is essentially more difficult than smaller degrees, and by the end of the course we'll understand this difficulty. Along the way we'll give a thorough study of field extensions, and using some tools from linear algebra, shed light on some ancient problems about the constructibility of various geometric objects.

Math 361: Complex Analysis
Instructor: Caroline Turnage-Butterbaugh
Time: 5a
Prerequisite: Math 321 or instructor permission

Behold the power and beauty of analysis in the complex plane! In this setting, if a function has one derivative, it has infinitely many derivatives. If two functions are differentiable in an open set and are equal in an arbitrarily small disc inside that open set, then the two functions are equal in the whole open set. If a function is bounded and differentiable everywhere then it must beâ€¦ constant. As Riemann stated in

1851, "In effect, if one extends these functions by allowing complex values for the arguments, then there arises a harmony and regularity which without it would remain hidden." This course is recommended for those considering graduate school as well as anyone who loved the nuances of line integrals in Math 210/211.

Problems of the Fortnight

This is a special issue, so there is no problem of the fortnight this week, but feel free to take a look at problems from past issues near the whiteboard or online.

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