Folded Ribbon Knots Advisor: Kate Hake Terms: Fall/Winter Prerequisites: None

Project Description:

There are various ways to define a knot, all of which capture the intuitive notion of a knotted loop. Classically a knot can be defined as a closed, non self-intersecting smooth curve embedded in Euclidean 3-space. Sometimes the arbitrary flexibility and lack of thickness in the classical theory of knots does not accurately depict the physical constraints of objects in nature. This inspires questions in the field of physical knot theory and models that seek to capture some of the physical properties.

In this project, we will focus on folded ribbon knots. To create a folded ribbon knot in 3-space, start with a long, rectangular piece of paper, tie a knot in it, and connect the ends. Then flatten the ribbon into the plane, origami style, with folds in the ribbon appearing only at the corners. When we model a folded ribbon knot, we view the knot as a polygonal knot diagram, as shown below.



Figure 1: This figure shows part of a folded ribbon knot and the corresponding polygonal knot diagram. If the two ends are connected a trefoil knot is formed.

We define the ribbonlength of a folded ribbon knot to be the ratio of the length of the ribbon to the width. An interesting open question we will investigate is to find the minimum ribbonlength for a given knot type. We will also study the connections of folded ribbon knots to grid diagrams of knot, used in the combinatorial formulation of knot Floer homology, and Mosaic knots, used to model quantum knots.