# Gooδσελλ Gαzεττε

*Carleton College* *May 10, 2019*

*Northfield, MN 55057* *The newsletter for the Carleton mathematics and statistics community* *Vol. 37, No. 16*

## Comps Talks

Come support your fellow classmates and friends at their Comps Talks next week! Groups comps talks will take place next Tuesday, May 14 and Thursday, May 16 in Olin 141. Take a look at what they'll be speaking about below, then be sure to stop by and support them while they demonstrate what they've learned; you're likely to learn something new yourself as well!

## Tuesday, May 14
## Olin 141

**Title:** Statistical Analysis in College Football
**Speaker:** Elliot Cahn, Dylan Rye, Christian Zaytoun, Noah Feldman
**Time:** 3:30pm

**Abstract:** Each winter, American college football becomes the center of debate in the American sporting world. Coaches, pundits and casual fans alike argue over topics such as which teams are the best in the nation or who is going to win a major game. While different official outlets generate top-25 rankings that are used to determine which teams get into the College Football Playoffs, these are hardly controversy-free. In this talk, we will discuss the opportunities and challenges of predicting college football games and examine five models used to estimate the ability of NCAA football teams. Pythagorean wins and random forests seek to make predictions for future games based on comparisons of prior box-score results. Bradley-Terry and eigenvector methods consider results as well as strength of schedule to evaluate performance and determine ability. After exploring each model's individual performance, we will combine these four models into an ensemble with the aim of better predicting the outcome of games, with special focus on bowl games.

**Title:** Jackknifing Biodiversity
**Speaker:** Nupur Bindal, Alief Moulana, Tim Schoch, Ned Wang
**Time:** 4:30pm

**Abstract:** In a rapidly wrought world with ecological shifts due to development and climate change, understanding the effects to biodiversity is crucial for the preservation of numerous species and the environment. But how does one exactly measure biodiversity? Is it as simple as counting the number of different species in a community or the evenness of a population? And how does one even tell if two species are really different in the first place? In this talk, we will define, examine, and discuss various measures of biodiversity. We will also analyze an interesting method that uses similarity matrices called *Diversity of Order q of the Community*, which takes into account how much one cares about species rareness or dominance (which gives the value of *q*). After giving various examples of this measure, including an application on microbial ecology using statistics, the talk will then proceed to discuss how dual numbers could be incorporated to further study the *diversity of order q of the community.*

**Title:** Stanley's Chromatic Tree Conjecture: Color by Numbers? or Polynomials?
**Speaker:** Patty Commins, Josh Gerstein, Kiran Tomlinson, and Nick Vetterli
**Time:** 5:30pm

**Abstract:** You may be familiar with the four color theorem, which states that given four colors, we can fill in any (reasonable) map so that no two bordering countries share a color. But how many such colorings are possible? Surprisingly, it turns out that the number of colorings of a graph with *n* colors is a polynomial in *n*. Moreover, we can represent each coloring of a graph with a monomial. We will explore the "infinite polynomial" yielded by summing these monomials over all colorings of the graph, culminating in a fascinating open problem: up to isomorphism, are trees uniquely described by the sum of their colorings? Cookies will be provided at the department's expense.

## Thursday, May 16
## Olin 141

**Title:** On the Knotting Probability of Random Equilateral Hexagons
**Speaker:** Aniruddha Nadiga, Clara Buck, Sean Gallagher
**Time:** 3:30pm

**Abstract:** Classically, a knot is defined as a simple closed curve in 3-space. However, in real-world applications, the flexibility of topological knots doesn't capture constraints imposed by physical properties. For this reason, we look at geometric knots, which are modeled by *n*-sided polygons. Using techniques from symplectic geometry, the space of equilateral hexagons can be parameterized by 6 measure-preserving action-angle coordinates. We use this parameterization to lower the theoretical upper bound on the knotting probability of randomly generated equilateral hexagons.

**Title:** Number Theory Without Numbers
**Speaker:** Andrew Biehl, Brianna Fitzpatrick, Dallas Keate, and Janna Wennberg
**Time:** 4:30pm

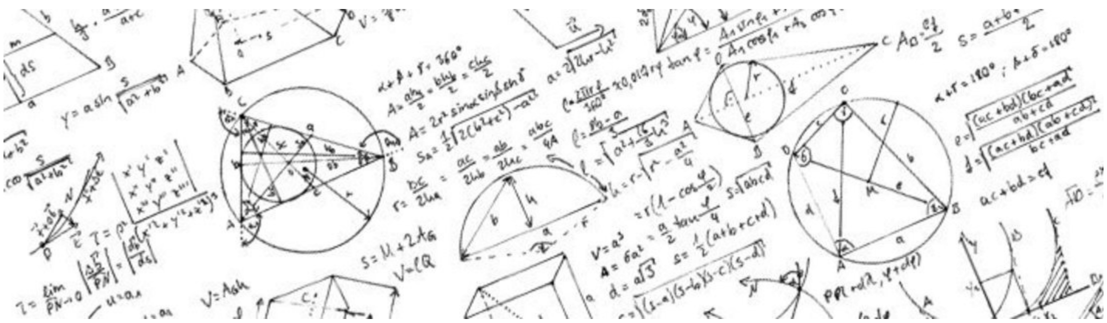
**Abstract:** Prime numbers have sparked important questions for mathematicians, including settled questions such as "How many prime numbers are there?" (answered by Euclid in 300 BC) and open questions such as the Riemann Hypothesis (posed by Riemann in 1859). Is it possible to reframe these number theoretic questions using different building blocks, like polynomials? It turns out you can! In our talk, we will give a brief overview of concepts from elementary number theory as well as their analogues in finite fields. From there, we can use these new tools to prove the Prime Number Theorem in this setting.

**Title:** Closure-Complement-Frontier Problems in Multitopological Spaces
**Speaker:** Sara Canilang, Nicolas Graese, Ian Seong
**Time:** 5:30pm

**Abstract:** Suppose we have a set, and we apply closure and complement operators. How many different sets can we get? The answer to this question turns out to be 14-- this is a well-known result from Kazimierz Kuratowski's 1922 thesis. Now, what if we apply different operators, such as frontier (boundary), intersection, or union? What if we include different topologies, other than the usual topology on the reals we are used to? In this talk, we will discuss the history and proof techniques of these problems and more. We'll employ both an algebraic perspective, studying monoids of operators (which are like groups without inverses), and a topological one, constructing some intricate subsets of the real line. Our main result is a solution of the closure-complement-frontier problem for a set equipped with two saturated topologies. We will also describe our efforts to generalize from two saturated topologies to an arbitrary number of saturated topologies.

# What's the Math and Stats Department Teaching Next Term?

Have you checked your registration number yet? Made a list of classes you're hoping to take next year? Let the course descriptions below guide you into an adventurous new school year within the Carleton Department of Mathematics and Statistics! There's something for everybody, from applied regression to advanced linear algebra -- find out more below.



**Math 207:** Communicating Mathematics
**Instructor:** Deanna Haunsperger
**Time:** Tuesdays 10:10 - 11:55, 2 credits (mandatory S/Cr/NC)
**Prerequisite:** None

When Aunt Hildegard asks you over the summer what you learned in Elementary Number Theory this spring, and you know she last studied math thirty-five years ago, what are you going to tell her? How should you argue to your US representatives that they should believe the mathematics showing your legislative districts have been gerrymandered? How would you write an editorial for the local paper to show that math is both beautiful and useful? We need to communicate mathematics all the time to non-mathematical audiences; in this course you'll get experience each week in writing or presenting some mathematical ideas to a non-mathematical audience and critiquing the work of others.

**Math 236:** Mathematical Structures
**Instructor:** Alex Barrios

**Time:** 2a
**Prerequisite:** Math 232 or permission of the instructor

What lies beyond calculus and linear algebra? How do we prove mathematical statements? How do we even think of possible statements, and what makes us suspect that a statement may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with deep roots that tap into the wisdom of many generations. Nevertheless, if you've ever wondered "How could anyone come up with that?", or "How can you really be sure of that?", about some mathematical result, taking this course may help dispel some of the mystery. We'll spend considerable time developing theorem-proving and problem-solving skills. Along the way, we'll take a new and closer look at some old friends, such as functions. In the final part of the course, we'll use functions to compare the "size" of various infinite sets. For example, we'll see that despite appearances, there are not any "more" rational numbers than there are integers; on the other hand, there are "more" real numbers than rational numbers. Math 236 is the first course in our curriculum whose primary goal is to teach you how to write proofs. Because proofs are how mathematical truth is established, it will give you the keys to a whole new mathematical world---not to mention the necessary prerequisite for a slew of upper-level mathematics courses.

**Math 245:** Applied Regression Analysis
**Instructor:** Katie St. Clair
**Time:** 2a
**Prerequisite:** Math 215 (or equivalent) or 275

Model building is a fundamental idea in statistics. In your intro stats class you learned some basic techniques for modeling a response as a linear function of one explanatory variable (simple linear regression). In this second stats course you will learn more advanced techniques for building regression models that can include many explanatory variables (multiple regression) or a categorical response (logistic regression). We will apply these techniques to explore how air pollutants might affect mortality, whether sex plays a role in determining a worker's salary, and how a regression model predicted a national tragedy. This course emphasizes model building and checking techniques and statistical writing. We will meet in a lab and use the free statistical software R. As the title suggests, this is an applied course so you will be working with new data sets each week, and you can expect to be a seasoned R user by the end of the term!

**Math 265:** Probability
**Instructor:** Katie St. Clair / Josh Davis
**Time:** 1a / 5a
**Prerequisite:** Math 120 or 211

If the "immortal monkey" randomly strikes keys on a keyboard for eternity, what is the probability that it will eventually produce the complete works of Shakespeare? If in a small town, out of 12 car accidents that occured in June 1986, four of them occured on Friday the 13th, would this confirm your hunch that "13" is unlucky? Probability is a fundamental branch of mathematics and is the foundation of all methods of statistical inference. In this course we will use the tools of counting and calculus to model random events, compute probabilities, and have lots of fun with balls in urns, poker hands, and coins and dice (fair or otherwise).

**Math 280:** Statistical Consulting
**Instructor**: Adam Loy
**Time:** Tuesday only, 2/3c
**Prerequisite:** Math 245 and permission of the instructor

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

**Math 295:** Numerical Analysis
**Instructor**: Rob Thompson
**Time:** 4a
**Prerequisite:** Math 232

This course will be a journey through the mathematical theory of a variety of numerical algorithms for answering practical computational questions. The two main themes will be linear algebra and optimization. We will learn about numerical methods for: understanding the geometry and solutions of a linear system, fitting functions and parameters to data, optimizing linear and convex functions with and without constraints, and more. Applications will be explored in class and via student driven projects. This course will involve both theory and implementation of numerical methods, so some past experience with a programming language (Python, Mathematica, R) is recommended. Come talk to Rob if you have questions!

**Math 315:** Bayesian Statistics
**Instructor**: Adam Loy
**Time:** 4a
**Prerequisite:** Math 275

For decades the world of statistics was dominated by "frequentist" methods. Bayesian statistics is an alternative school of thought founded upon the idea that our beliefs about the world are constantly revised with the incorporation of new information. While this idea is intuitive, Bayesian statistics was held back by the mathematical intractability of common inferential tasks. Computers have changed that. Today, Markov chain Monte Carlo (MCMC) methods are used by Bayesians to conduct statistical inference. In this course, we will explore the Bayesian philosophy and approach to statistical inference. We will start with the basic building blocks of inference and then explore common statistical models including regression and hierarchical models. Along the way, we will learn how and why MCMC works so that we can simulate from posterior distributions so that we are not restricted to models that can be "fit" using pencil and paper.

**Math 321:** Real Analysis I
**Instructor**: Mike Cohen
**Time:** 5a
**Prerequisite:** Math 236 or permission of the instructor

The invention of calculus, so-named for its power as a computational tool, is probably the most significant and widely applicable development in the history of mathematics. Its fundamental idea is the culmination of "infinite processes" via differentiation, integration, and series. But a free-wheeling approach to the methods of the calculus swiftly leads to questions that are difficult to resolve, or even mathematical paradoxes: Does every sequence of continuous functions converge to a continuous function? Can every power series be differentiated term by term? Is it true that 1-1+1-1+... = 1/2? It may surprise you to learn that mathematicians have not always agreed on the answer to these questions.

In this course, with an eye toward the difficult problems of the calculus, we will take a much deeper look at a foundational concept: the limit. The limit, often covered in a few short weeks in an introductory calculus course, is a sophisticated and subtle idea that was actually several hundred years in the making. We will practice many proofs with limits, and gain an intuition for the vital role they play in the study of functions of a real variable. We will also learn some things about the history of mathematics along the way. This course is highly recommended for anyone who is considering graduate school in math or statistics.

**Math 332:** Advanced Linear Algebra
**Instructor**: Mark Krusemeyer
**Time:** 3a
**Prerequisite:** Math 236 or permission of the instructor

Although you have seen linear transformations, matrices, determinants and vector spaces in Math 232, after a bit of review we'll quickly go beyond the material of that course as we investigate those concepts in more depth. For example, suppose there is no diagonal matrix fo a linear transformation (either because some of the eigenvalues aren't real, or because an eigenvalue is repeated and there aren't "enough" eigenvectors for it). How nice a matrix can you still get, if you insist on real entries? How about if your original matrix has rational entries, and you insist on rational entries? We'll do some "complex" linear algebra as well, likely including the study of unitary transformations and Hermitian matrices, and as time permits we may look at such topics a infinite-dimensional vector spaces (important in physics) and quadratic forms (which show up in several parts of mathematics, including topology). There are definitely more good topics than we will have time for, and you will likely end up exploring some topic with a partner instead of taking a final exam.

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## An Online Class You Might Be Interested In

**LACOL:** Operations Research
**Instructor:** Steven J. Miller (Williams College)
**Time:** MWF 9:00 am-9:50 central time, plus asynchronous options
**Prerequisite:** Linear algebra (programming experience and analysis are desirable, but not necessary)

**Abstract:** In the first N math classes of your career, you are sadly misled as to what the world is truly like. How? You're given exact problems and told to find exact solutions. The real world is sadly far more complicated. Frequently we cannot exactly solve problems; moreover, the problems we try to solve are themselves merely approximations to the world! We are forced to develop techniques to approximate not just solutions, but even the statement of the problem! In this course we discuss some powerful methods from advanced linear algebra and their applications to the real world, specifically linear programming and if time permits random matrix theory. Contact Adam Loy at aloy@carleton.edu if you are interested.

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## Problems of the Fortnight

This is a special issue, so there is no problem of the fortnight this week, but feel free to take a look at problems from past issues near the whiteboard.

*Editors:* ***Saahithi Rao, Owen Biesel***
*Problems of the Week:* ***Mark Krusemeyer***
*Web & Subscriptions:* ***Sue Jandro***