Gooδσελλ Gazεττε

Carleton College | February 15, 2019
Northfield, MN 55057 | The newsletter for the Carleton mathematics and statistics community | Vol. 37, No. 10

Comps Talks

Come support your fellow classmates and friends at their Comps Talks next week! Students who elected to pursue an independent comps project will present their findings on Tuesday, February 19 in CMC 206. Groups comps talks will take place the following Thursday on February 21 in Olin 141. Take a look at what they'll be speaking about below, then be sure to stop by and support them while they demonstrate what they've learned; you're likely to learn something new yourself as well!

Tuesday, February 19
CMC 206

Title: Bezout's Theorem & Algebraic Geometry
Speaker: Noah Goldman
Time: 3:30pm

Abstract: Anyone familiar with systems of polynomial equations (whether they majored in math or just had to solve a train-related word problem once) knows solving them can get complicated. Even when we can find solutions, who knows if there are more? How many ought we to expect? The answer to this question, Bezout's theorem, lies at the "intersection" of geometry and algebra. To address the *algebraic* problem presented by a polynomial system, Bezout's theorem solves the equivalent *geometric* problem of finding points of intersection between hypersurfaces, and vice versa --- simultaneously generalizing both the fundamental theorem of algebra and the concept of linear independence. To prove it, we will embark on a whirlwind tour through algebraic geometry, exploring projective sets, homogeneous ideals, Hilbert polynomials, and the great *Nullstellensatz*, guided only by that simplest of links between pictures and equations: the coordinate.

Title: De Rham Cohomology
Speaker: Charlie Kapsiak
Time: 4:00pm

Abstract: There exists a variety of ways to algebraically examine the topology of space. When the space has a differential structure, one such method is furnished by the de Rham cohomology. In this talk, we examine the construction of the de Rham cohomology. Further we motivate and prove a remarkable result: the de Rham cohomology, despite being constructed via the local smooth structure, is actually a topological invariant.

Title: Factor Analysis
Speaker: Xiao (Marshall) Ma
Time: 4:30pm

Abstract: Sometimes we do not have the precise data that we need for an answer, say when a psychologist tries to draw the connections between students' "mental resilience" and their "cognitive abilities." Though we cannot observe these two qualities directly, we can pull data that are loosely related to these categories and extrapolate relationships between the two categories via factor analysis. In this talk specifically, I will explore the factor analysis model, introduce methods for estimating the model, and explain the concept of factor rotation that facilitates our approach to find the best factors. In the end, I will apply these methods to a real-world dataset and show what insights factor analysis can bring.

Title: Graph colorings and the four-color problem
Speaker: Haoyi Wang
Time: 5:00pm

Abstract: How many colors do we need to properly color the world map? Can you believe we only need 4 colors to do so? This is the famous four color problem. We can actually solve this problem using graph theory, where we color vertices instead of countries. Now, if we were to color the edges instead of vertices, how would the problem change? If each vertex is adjacent to three edges, are three colors enough to color all the edges so that two edges that are adjacent to the same vertex do not have the same color? Sadly, this is not always the case. In this presentation, I will talk about non-3-colorable graphs, how the famous four color problem and the three edge-coloring problem are related, and how we can translate the four color problem into other interesting problems.

Title: Integral Apollonian Circle Packings
Speaker: Samantha Kile
Time: 5:30pm

Abstract: This presentation is about circle packings, with a focus on integral Apollonian circle packing, their applications and 3D examples. A circle packing is when tangent circles are used to fill the space, in particular another circle. Integral Apollonian circle packings are a special type where all the circle have an integer curvature. I will present how to make tangent circles using only a compass and a straight edge, how to construct Apollonian integral circle packings and talk about the features such as interesting layouts of the interior circles.

Title: Phase Response Curves in Neuroscience
Speaker: Keenan Ronayne
Time: 6:00pm

Abstract: To a mathematician, a periodically firing neuron provides a perfect example of a dynamical system. In this sense, neurons are limit cycle attractors with periods matching their period of oscillation. Many intrinsic characteristics of neurons are gleaned from this formulation, including phase response curves (PRCs). PRCs measure the shift in phase of a neuron due to brief external stimulation. In phase space, a perturbation is a deviation from the limit cycle, which either increases or decreases the time to complete the subsequent cycle. PRCs are helpful in modeling weakly coupled oscillators, where the perturbation received by a neuron is from the impulse of an adjacent neuron. The interaction between two neurons is often estimated by their voltages, but a much simpler representation can be found using the neurons' relative phases. Taking this a step further, networks of multiple neurons can be modeled to uncover how changes in individual neuron PRCs affect the dynamics of the network. Thus, phase response curves bridge the gap between individual and network level dynamics, a central theme in contemporary neuroscience research.

Thursday, February 21
Olin 141

Title: A Statistical Analysis of Changes in Extreme Precipitation in the Coastal Carolinas
Speaker: Pedro Girardi, Trevor Freeland, Joseph Nardi
Time: 3:30pm

Abstract: Extreme weather events can be destructive to communities around the world. Climate scientists have been studying the potential changes in extreme precipitation stemming from anthropogenic climate change. In recent years, extreme precipitation events such as Hurricane Florence in the coastal Carolinas have further motivated such studies. In order to address such questions from a statistical perspective, one ought to use models specifically designed to analyze extreme events. We discuss the use of the generalized extreme value (GEV) distribution as a model for yearly maximum precipitation. Using a GEV model, we analyze historical precipitation data in the coastal region of North and South Carolina to estimate historical changes in the distribution of precipitation extremes potentially caused by anthropogenic climate change. Our results indicate that events at least as extreme as Hurricane Florence in the coastal Carolina region were, because of anthropogenic climate change, 35.2% (95% confidence lower bound: 7.9%) more likely in 2018 than they were in 1950. Our historical analysis provides one piece of evidence of anthropogenic climate change influencing extreme precipitation events, which motivates the continued study of changes in extreme precipitation using historical data as well as dynamic climate models.

Title: Koch-Like Curves
Speaker: Robert Browning, David Perl, Daniel Tsui, Terry Wang
Time: 4:30pm

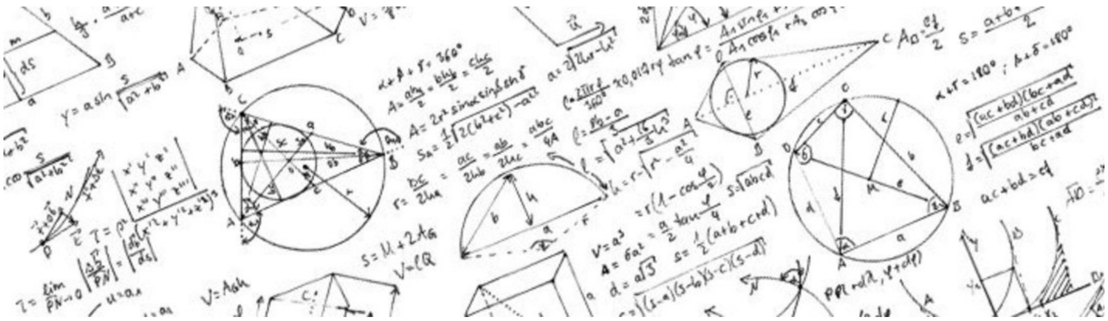
Abstract: Fractal images have fascinated people for years, presenting a way to visualize the beauty of math. One of the simplest fractals is the famous Koch Snowflake, which can be drawn using a recursively generated sequence of instructions. We examine variations of these instructions. Our goal is to characterize how changing the original sequence of angles affects the resulting image. Images produced in this manner vary from ever-growing tangles to highly structured and symmetric patterns. In particular, we investigate the conditions for rotational symmetry, as well as calculate the order of rotational symmetry for an image. In our talk, we explain how these images are generated. We also provide some of our favorite images and explore their interesting properties.

Title: Spatial Statistics
Speaker: Yuta Baba, Matt Carter, Saahithi Rao
Time: 5:30pm

Abstract: What happens when the data at one location are associated with the data occurring at a location nearby? We will investigate this question and what it means to account for spatial dependency. Specifically, we will be looking at Airbnb locations in Boston, mercury levels in floodplains of South River in Virginia, and influenza outbreaks in Colorado.

What's the Math and Stats Department Teaching Next Term?

Have you checked your registration number yet? Made a list of classes you're hoping to take next term? Let the course descriptions below guide you into an adventurous new year and new term within the Carleton Department of Mathematics and Statistics! There's something for everybody, from applied regression to combinatorial games -- find out more below.



Math 236: Mathematical Structures
Instructor: Mark Krusemeyer
Time: 2a and 5a
Prerequisite: Math 232 or permission of the instructor

What lies beyond calculus and linear algebra? How do we prove mathematical statements? How do we even think of possible statements, and wha makes us suspect that a particular statement may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with dee roots that tap into the wisdom of many generations. Nevertheless, if you've ever wondered "How could anyone come up with that?", or "How can you really be sure of that?", about some mathematical result, taking this course may help dispel some of the mystery. We'll explore various concepts, especially from set theory, that are indispensable for most areas of advanced mathematics, and we'll spend considerable time developing theorem-proving and problem-solving skills. Along the way we'll take a new and closer look at some old friends, such as functions and relations: What are they really? In the final part of the course we'll use functions to compare "size" of various infinite sets. For example, we'll see that despite appearances, there are not any "more" rational numbers than there are integers; on the other hand, there are "more" real numbers than rational numbers. If you're considering a math major, taking this course should help you decide; also, "Structures" is a prerequisite for the majority of upper-level math courses.

Math 241: Ordinary Differential Equations
Instructor: Mike Cohen
Time: 3a
Prerequisite: Math 232 or permission of the instructor

An ordinary differential equation is an equation which describes a relationship between an unknown function, its dependent variable, and one or more of its derivatives. Setting up differential equations and finding solutions is of vital importance not only in mathematics but also in modeling problems which arise in physics, biology, economics, and engineering.

Students who enjoy calculus techniques (especially methods of integration) will have fun computing solutions via classical methods and ad hoc trickery. In our efforts to understand as many types of differential equations as possible, we'll also develop a theoretical understanding of vector spaces of solutions, and make robust use of concepts from linear algebra like eigenvalues and eigenvectors. We will even make some brief forays into the world of complex numbers, and explore other concepts of broad interest, depending on student preferences.

Math 245: Applied Regression Analysis
Instructor: Adam Loy
Time: 4a
Prerequisite: Math 215 (or equivalent) or 275

Does smoking cause cancer? How do we know this? Is there a gender wage gap after controlling for education and experience? How is this quantitative argument made? In our first statistics course we focus on modeling the relationship between a response variable and a single predictor; however, many questions cannot be answered using such models. In this course, we will explore how to incorporate multiple predictors into our models to answer complex questions. In addition to learning how to model continuous response variables, we will explore models for binomial and Poisson counts. This course emphasizes model building, model validation, and how to clearly communicate the results of our models. We will make frequent use of R.

Math 255: Introduction to Sampling Techniques
Instructor: Katie St. Clair
Time: 2a
Prerequisite: Math 215 or 275

This course covers a wide range of statistical sampling techniques that are used to make inferences about a population. We will discuss how to form estimates and quantify the sampling error using "sampling weights" when data is collected using sampling designs that are more complex than a simple random sample. We will also cover strategies for determining an "optimal" sampling design when resources (time/money) are limited. Time permitting, we may also cover how sampling weights are used in data visualization, regression models or chi-square hypothesis tests. Applications will be drawn from both the natural and social sciences, and we will use the R survey package extensively throughout the course.

Math 261: Functions of a Complex Variable
Instructor: Caroline Turnage-Butterbaugh
Time: 4a
Prerequisite: Math 210 or 211

In introductory calculus, we study certain special functions (polynomial, trigonometric, exponential, etc) through the lenses of limits, derivatives, and integrals. While the material covered in three terms of calculus is expansive, there is an ever present, underlying connective thread: the functions we study are functions of a real variable, meaning their domain is always some subset of the real numbers. What if we start the conversation over and let the domain of the functions consist of complex numbers? How do we perform calculus (limits, derivatives, integrals) with a function of a complex variable? What do the plots of these "extended" functions even look like? We will investigate these questions and even find real-world applications to these questions in Math 261.

Math 275: Statistical Inference
Instructor: Andy Poppick
Time: 5a
Prerequisite: Math 265

Statistics is the discipline concerned with how data are used to make inferences about populations or processes exhibiting variability, and how to quantify uncertainty in those inferences. In this course, we develop tools to evaluate what we know and don't know about the observed world. We will introduce some of the theory behind inferential methods, using the language of probability, and we'll also learn to apply these methods in realistic settings. An additional emphasis will be placed on computational tools for data analysis, using R.

Math 280: Statistical Consulting
Instructor: Andy Poppick
Time: Tuesday only, 2/3c
Prerequisite: Math 245 and permission of the instructor

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

Math 295: Combinatorial Games
Instructor: Eric Egge
Time: 2/3c
Prerequisite: Math 236 or permission of the instructor

A combinatorial game is a two-player game in which players alternate moves, there are no random devices like dice, and both players always know everything about the current state of the game. Examples that might be familiar include Chess, Checkers, Dots and Boxes, and Tic Tac Toe. Games that are not combinatorial games include Battleship, Risk, and Monopoly. Combinatorial games are different from the games that arise in Economics, which often involve payoffs, and whose study is sometimes called Game Theory.

In this course we will study both the theory and practice of combinatorial games. On the theory side, we will develop a way to completely solve impartial games, which are games in which both players always have the same moves available. The canonical impartial game is Nim. We will then study partizan games, in which players might not have the same moves available as their opponent has. Chess is a partizan game: if my opponent can move the white bishop on her turn, then I am not allowed to move the white bishop on my turn, so we have different moves available to us. Here we will find we can use combinatorial games to reconstruct the real numbers, but now with additional bells and whistles, including infinitesimals. We will pepper the theory with the practice, by studying a variety of specific combinatorial games. These might include Nim, Domineering, Amazons, Chomp, the octal games, and variations of these games, but there are numerous other possibilities.

This class will include a substantial amount of group work, and students presenting ideas, conjectures, and solutions at the board. In addition, this course is new, and it might require adjustments as the term goes on. So if you're looking for a predictable, carefully-planned course in which you can mostly sit back and listen, then you might want to look at other courses. But if you're looking for a class where you can have fun bonding with your classmates over your enjoyment of (or occasional frustration with) math and games, and you don't mind a little unpredictability, then this class might be for you.

Math 312: Elementary Theory of Numbers
Instructor: Caroline Turnage-Butterbaugh
Time: 5a
Prerequisite: Math 236 or permission of the instructor

The theory of numbers is primarily concerned with understanding the properties of the natural numbers and, more specifically, the primes. With such a broad topic at hand, it should not be too surprising that much is known and much is unknown. Some questions were asked and answered long ago. For example, Euclid (300BC) proved that yes, there are infinitely many primes. Some questions were asked and answered not too long ago. For example, it was only recently (2013) that Helfgott proved the Ternary Goldbach Conjecture (posed in the 1700s), that every odd number greater than 5 can be written as the sum of 3 primes. Some questions were asked and still remain unsolved! For example, we do not know how to prove the Twin Prime Conjecture, that there are infinitely many pairs of primes whose difference is exactly 2. In this course, we will study the numbers with "elementary" tools, which roughly equates to "tools developed before the invention of calculus." We will encounter the Euclidean algorithm, prime factorization, multiplicative functions, congruences, and quadratic reciprocity.

Math 331: Real Analysis II
Instructor: Gail Nelson
Time: 4a
Prerequisite: Math 321 or permission of the instructor

Now that you have mastered the ideas behind epsilon-delta proofs it is time to take advantage of your new skills. In this course we will look closely at the notion of integration. After a review of Riemann integration we will turn our attention towards other topics. More specifically, this course includes Lebesgue measure, the Lebesgue integral, an introduction to general measure theory, and Banach and Hilbert spaces. Not only is this your chance to "integrate" your knowledge of functions, it is also an opportunity to better your understanding of the legal interchange of limit operations. And if you know the instructor, it shouldn't surprise you if the Cantor set makes an appearance! The flavor of the course will be similar to a graduate-level course in analysis. If there is a possibility that you are headed for graduate school in mathematics or a related field, this course comes highly recommended.

Math 341: Partial Differential Equations
Instructor: Rob Thompson
Time: 3a
Prerequisite: Math 241

About 200 years ago, Jean Baptiste Fourier studied the way that heat moves through a flat metal plate via a partial differential equation called the heat equation. Trying to describe his observations mathematically, he did a seemingly simple thing: he expressed the heat distribution as a sum of sines and cosines (a "Fourier series"). Expressing the complicated behavior of heat in terms of simpler functions gave Fourier powerful insight into the behavior of the heat equation. Fourier's idea revolutionized pure and applied mathematics.

In this course, we'll learn the fundamentals of partial differential equations and make a tour of Fourier's revolution. We'll examine various interesting PDE (including the heat equation) and their applications to wave propagation, heat conduction, elastic equilibrium, and more. We'll also develop ideas from Fourier analysis as needed to access information about the solutions to the PDE we study. Feel free to contact me (rthompson) with any questions!

Math 342: Abstract Algebra I
Instructor: Owen Biesel
Time: 2a
Prerequisite: Math 236 or permission of the instructor

Abstract algebra grew from the study of number systems beyond the familiar integers and real numbersÃ¢â,¬â€are there other ways to add extra numbers besides including the complex ones? What about "clock arithmetic" in which 0 and 12 are the same? We'll start with groups, the abstract axiomatization of symmetries and other undoable transformations: from a tiny list of straightforward axioms, we will derive a rich theory of the structure of permutations, matrices, and the principles that let us count configurations like Sudoku grids or plane tessellations "up to symmetry." In the last phase of the course we will study rings (generalizations of the integers) and fields (generalizations of the real numbers) and show that many familiar facts about numbers are rather fragile, while others extend far beyond their original setting.

This class is recommended for anyone who likes things to feel exactly true, not just true in the limit; anyone who wants to get to know what kinds of mathematical structures exist "out there"; and anyone who is also interested in art, theoretical physics, or the rest of mathematics.

Another Class You Might Be Interested In

Philosophy 236: Proof, Knowledge, and Understanding in Mathematics
Instructor: Douglas Marshall
Time: 4/5c
Prerequisite: None (but see description)

Abstract: An introduction to the philosophy of mathematics focusing on the history and development of mathematical proofs. The course is organized around three central questions: i. What is the relationship between a mathematical proof and our knowledge of the theorem that it proves? ii. Do some mathematical proofs go beyond establishing the truth of their theorems and actually explain why the theorems are true? iii. How has our mathematical knowledge grown throughout history? We will first address these questions by reading and discussing Imre Lakatos's book Proofs and Refutations. We will continue with readings drawn from classic and contemporary sources in the history and philosophy of mathematics. This course has no formal prerequisites, though it does presuppose a willingness to read, assess, and write about mathematical proofs.

Kolenkow-Reitz Fellowship Information Session

The Kolenkow-Reitz information session will be held in Hulings 120 on February 20th at 4:30 PM. The Kolenkow-Reitz fellowship provides research support for Carleton students working with non-Carleton science and math faculty at another institution during the summer break. These research opportunities are intended to encourage Carleton students' development as researchers and their exploration of mathematics and the sciences as a possible career. Awards fund student stipends ($470/week for full time work) for up to 10 weeks. This year's application deadline is March 15th. Here's a link to more information about Kolenkow-Reitz: https://apps.carleton.edu/mathscience/faculty/studentresearchaway/.

AASHE Conference

The Association for the Advancement of Sustainability in Higher Education, AASHE, Conference will be held in Spokane, Washington October 27-29th, 2019. The theme of the conference is: Co-Creating a Sustainable Economy. AASHE 2019 is tackling the root cause for the continued rise in carbon emissions: our dysfunctional economic system. The conference seeks to showcase and strengthen higher education's contributions to the movement for a sustainable economy, which we see as inclusive of the exciting work happening under a variety of other names such as the solidarity economy, wellbeing economy, circular economy, post-growth economy, regenerative economy and restorative economy. This is a great opportunity for students to network with thousands of attendees representing hundreds of institutions from around the nation. The Environmental Advisory Committee sponsors and funds one to two students to attend this conference each year. Note that applications should be completed by February 22 to be considered. To apply, visit: https://apps.carleton.edu/governance/environment/.

Problems of the Fortnight

This is a special issue, so there is no problem of the fortnight this week, but feel free to take a look at problems from past issues near the whiteboard.

Editors: ***Saahithi Rao, Owen Biesel***

Problems of the Week: ***Mark Krusemeyer***

Web & Subscriptions: ***Sue Jandro***