Special Edition: Northfield Undergraduate Mathematics Symposium

Date: Thursday, October 13
Location: CMC 206

Each year Carleton and St. Olaf students work on a variety of interesting research problems in mathematics, both here in Northfield and around the country. Several of these students will be sharing the work they did this past summer at the 2016 Northfield Undergraduate Mathematics Symposium. Please join us for as many of the talks as you can attend, as well as for a pizza dinner. These talks all promise to be fascinating in their own right, but each one also counts for half of the eight talks junior and senior math and math/stats majors need to attend to satisfy their lecture attendance requirement. Below is a schedule for the symposium, followed by the titles and abstracts for the talks.

Schedule of Talks

3:40 – 4:00 pm  m-gapped Progressions and van der Waerden Numbers
Daniel Lewitz, Carleton College

4:05 – 4:25 pm  An Involusion Proof of a Borwein Theorem for Overpartitions
Conrad Parker and Melanie Stevenson, St. Olaf College

4:30 – 4:50 pm  A Combinatorial Model of Quantum Skew Symmetric Matrices
Eleanor Campbell, Carleton College

4:55 – 5:15 pm  Partial Differential Modeling in the Kidney
Quinton Neville, St. Olaf College

5:20 – 5:55 pm  Dinner (will be provided)

6:00 – 6:20 pm  Finding Minimal Spanning Forests in a Graph
Abdel-Rahman Madkour and Philip Nadolny, St. Olaf College

6:25 – 6:45 pm  The Metric s-t path Travelling Salesperson Problem and the
Randomized Christofides Algorithm
Shatian Wang, Carleton College

Titles and Abstracts

Title: m-gapped Progressions and van der Waerden Numbers
Speaker: Daniel Lewitz, Carleton College
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Abstract: An m-gapped progression is a generalization of an arithmetic progression in which the gaps in the progression only need to belong to a set of m elements, rather than all be the same. In the same way that arithmetic progressions pertain to the van der Waerden numbers, \( W(k; r) \), m-gapped progressions pertain to a function we call \( B_m(k; r) \). We will examine some results about the nature of the function \( B_m(k; r) \), both in general and for special case when \( r = 2 \). In particular, we will show how there are exact results for \( B_m(k; r) \) when \( k \) is relatively small. This work was done with Catherine Cooper, Trinity College; Alex Stoll, Clemson University; and Bruce Landman, University of West Georgia.

Title: An Involution Proof of a Borwein Theorem for Overpartitions
Speakers: Conrad Parker and Melanie Stevenson, St. Olaf College

Abstract: In 1990, P. Borwein conjectured a + - - sign pattern for polynomials counting certain signed integer partitions. We conjecture a + - 0 pattern for the generating function for overpartitions into parts not divisible by 3 and give an involution-based proof of the 0 case of this conjecture using pentagonal numbers and the Jacobi triple product. We also share a proof of a generalization of this case involving quadratic nonresidues modulo a prime.

Title: A Combinatorial Model of Quantum Skew Symmetric Matrices
Speaker: Eleanor Campbell, Carleton College

Abstract: The quantized coordinate ring of \( m \times n \) quantum matrices, or simply quantum matrices, holds deep connections to the theory of totally nonnegative matrices, wave interactions and knot theory. We examine the less understood theory of quantum skew-symmetric matrices \( O_q(S_{k,n}) \) over a field \( k \). This algebra is known to be generated by a set of generators \( y_{ij} \), \( 1 \leq i < j \leq n \), which satisfy certain commutativity relations dependent on some element \( q \in k \). We view \( O_q(S_{k,n}) \) from a combinatorial perspective. We prove \( O_q(S_{k,n}) \) is isomorphic to an algebra called \( A_k \) over \( k \), defined graphically. \( A_k \) is generated by elements \( x_{ij} \), where each \( x_{ij} \) is the sum of the weights of paths from \( i \) to \( j \) in a particular directed graph. The weights are obtained from elements of a space with simpler commutativity relations dependent on \( q \). Using inductive methods on the graph, we prove that the generators of \( A_k \) satisfy the same commutativity relations of \( O_q(S_{k,n}) \), allowing for a new combinatorial perspective that may be used to study this algebra.

Title: Partial Differential Modeling in the Kidney
Speaker: Quinton Neville, St. Olaf College

Abstract: The kidney, a diverse and complicated biological system, is most simply charged with the production of urine. At a deeper level, the functional unit that facilitates this process is the nephron. The nephron utilizes the Tubuloglomerular Feedback (TGF) System to monitor chloride levels during the production of urine, making sure the body is not retaining or losing too much. In mammals, there are two types of nephrons: short-looped and long-looped. The key difference between the short-loop and the long-loop is the existence of the Thin Ascending Limb (THAL) in the long-loop, which has differing properties for spatially varying permeability and maximum transport rate of chloride. Additionally, there is biological evidence of an association between desert mammals' ability to produce more highly concentrated urine and a higher percentage of long-looped nephrons in their kidneys. The long-looped nephron, however, is not well characterized biologically, which provides motivation to attempt to explain this association mathematically. Thus, we developed a partial differential model of a long-looped nephron, derived a characteristic ordinary differential equation, varied the length of the model THAL, and performed a bifurcation analysis of the long-looped TGF system. Our analysis indicates that there is a higher tendency towards oscillatory solutions in the long-looped TGF system than in the short-looped, and increasing the length of the THAL may create a more stable TGF system.
Title: Finding Minimal Spanning Forests in a Graph
Speakers: Abdel-Rahman Madkour and Philip Nadolny, St. Olaf College

Abstract: In the computation of multidimensional persistent homology, a popular tool in topological data analysis, a family of planar graphs arises. We have studied the problem of partitioning these graphs in a way that will be useful for parallelizing the persistent homology calculation. Specifically, we desire to partition an edge-weighted, undirected graph \( G \) into \( k \) connected components, \( G_1, \ldots, G_k \). Let \( w_i \) be the weight of a minimum spanning tree in component \( G_i \). For our purposes, an ideal partition is one that minimizes \( \max \{w_1, \ldots, w_k\} \). This problem is known to be \( \text{NP} \)-hard in the case of general graphs and we are unable to find this specific problem in the graph partitioning literature. We propose two approximation algorithms, one that uses a dynamic programming strategy and one that uses a spectral clustering approach, that produce near-optimal partitions in practice on a family of test graphs. We present detailed descriptions of these algorithms and the analysis of empirical performance data.

Title: The Metric s-t path Travelling Salesperson Problem and the Randomized Christofides Algorithm
Speaker: Shatian Wang, Carleton College

Abstract: In the well-known metric Travelling Salesman Problem (metric TSP), a complete graph \( G = (V, E) \) is given with nonnegative metric edge costs. The goal is to find a Hamiltonian circuit in \( G \) with minimum cost. The Christofides heuristic (1976) gives a nice purely combinatorial 1.5-approximation algorithm for the metric TSP. An important variant of the metric TSP is the metric s-t path TSP, in which two fixed vertices, \( s \) and \( t \) are given, and the goal is to find a min-cost Hamiltonian path from \( s \) to \( t \). The Christofides heuristic can be easily extended to this s-t path variant, but only with an approximation ratio of \( 5/3 \). An, Kleinberg and Shmoys (2012) made the first improvement to \( 5/3 \) with an LP based algorithm, the randomized Christofides algorithm, and achieved an approximation ratio of 1.618. This LP based analysis has inspired a sequence of later breakthroughs, including an 1.6 bound by Sebo (2013) and an 1.566 bound by Gottschalk and Vygen (2016). If time permits, a more general problem, the connected T-join problem, will be discussed at the end of the talk.