Mathematical expressions

2. Roman versus italic type

(1) According to conventional practice, Latin letters used as mathematical symbols are set in italic type to distinguish them from ordinary roman text...

(2) Boldface is used for three-vectors, dyadics, some matrices, tensors without indices, etc. It is inappropriate for four-vectors (*k*), vectors represented by a typical component, such as x_i for x = (x_1, x_2, x_3), and the magnitude of a vector, *H* for **H**. Subscripts attached to a three-vector should not be boldface unless they would be so as main characters.

When it is essential to distinguish between vectors and tensors, sans serif may be used for tensors.

C. DISPLAYED EQUATIONS

Display all numbered and complicated unnumbered equations on separate lines set off from the text above and below. "Complicated" equations are equations hard to accommodate within running text. These include equations longer than about 25 characters,

$$\mathrm{F_o} = \mathrm{N_1}\left[\mathrm{f}\left(\frac{\mathrm{C}}{(\mathrm{n})}\right) + \mathrm{K}(\mathrm{C}(\mathrm{n}))\mathrm{C}(\mathrm{n}^2) + \ldots\right] \tag{1}$$

and equations that contain built-up fractions, matrices or matrix like expressions, multilevel indices, or integral, summation, or product signs with limits:

$$\text{á} = \frac{\mathrm{B}(\mathrm{E_o}) + \mathrm{c}}{\mathrm{f_1} + \mathrm{f_2}}\mathrm{f}(\text{ù}) \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{2}$$

$$\hat{\beta} = \sum_{i=1}^{n} P_i$$

1. Multilinear equations

(1) If a displayed equation will not fit on one line, break the equation according to the following models:

$$\begin{aligned} \langle \mathrm{k} + \mathrm{q}|\mathrm{V_o}|\mathrm{k}\rangle = {} & \langle \mathrm{k} + \mathrm{q}|\mathrm{V}(\mathrm{r}, \mathrm{r_o})\text{ó}(\mathrm{r_o}, \mathrm{r})\mathrm{d}\text{ó}|\mathrm{k_o}\rangle_o \\ & + \langle \mathrm{k} + \mathrm{q}|\mathrm{v}(\mathrm{r} - \mathrm{r_o})\text{ó}(\mathrm{r}, \mathrm{r_o})\mathrm{d}\text{ô}_\mathrm{o}|\mathrm{k_o} + \mathrm{q}\rangle \\ & \times \langle \mathrm{k} + \mathrm{q}|2\mathrm{N_1}|\mathrm{k_o}\rangle + \langle \mathrm{k} + \mathrm{q}|2\mathrm{N_2}|\mathrm{k_o}\rangle \end{aligned} \tag{3}$$

$$\begin{aligned} & (\mathrm{h}\text{Ã}_{\mathrm{nk}} - \mathrm{h}\text{ù}_\mathrm{c})\langle \mathrm{nk}|\mathrm{f}|\mathrm{n} + 1, \mathrm{k}\rangle \\ & \quad = 2\mathrm{le^n E_o}(1 - \mathrm{e}^{-\mathrm{s}})\left(\frac{\mathrm{n}}{2}\right)^{1/2}(\mathrm{n} - 1)^{1/2}\mathrm{f_{nk}} \end{aligned} \tag{4}$$

Operators and signs of relation begin continued lines. When a product is continued from line to line, as in Eq. (3), start the continuation with a multiplication sign.
(2) Align signs of relation in

$$\begin{aligned} R &= (2\partial)^{n/4} |\div|^{-1/4} \exp\left(-\frac{1}{4} \div_{ij} \hat{q}_i \hat{q}_j\right) \\ &= \frac{1}{2} a_{ij} \hat{q}_i \hat{q}_j + b_1 \hat{q}_i + c \end{aligned} \tag{5}$$

and

$$\begin{aligned} S(k,0) &\approx 1 - \bar{ç}^2, k \neq 0 \\ &\approx 0, k = 0 \end{aligned} \tag{6}$$

or, alternatively,

$$\begin{aligned} S(k,0) &\approx 1 - \bar{ç}^2 \qquad (k \neq 0) \\ &\approx 0 \qquad (k = 0) \end{aligned} \tag{7}$$

2. Equation numbers

(1) Only displayed equations may be numbered.

(2) The preferred style is to number equations consecutively throughout the text with arabic numerals in parentheses: (1), (2), (3), etc. Numbering by section is also acceptable, if the section number is made part of the equation number: (2.1), (2.2), (2.3), etc. In appendixes use the numbering sequence (Al), (A2), (A3), etc.

(3) Place equation numbers flush with the right margin. Leave a space at least two characters wide between an equation and its number.

(4) An equation number should be centered beside a group of equations identified by one number, as in Eqs. (2), (6), and (7) on this page. It should be aligned with the last line of a multilinear equation, as in Eqs. (3)-(5).

3. Built-up fractions, slashed fractions, and negative exponents

(1) A fraction can be represented in three alternative ways: "built up" with a fraction line,

$$\frac{x + y}{z}$$

or "slashed" with a solidus,

$(x + y)/z$,

or with negative exponents,

$(x + y)z^{-1}$.

In running text one of the last two forms must invariably be the choice, since built-up fractions will not fit. In displayed equations, use the slashed form or negative exponents for short, simple fractions, but use the built-up form for fractions that are long and complex. That is, write

$$pq^2/r + (n/y)^2 = 0$$

or, better,

$$(pq^2/\mathrm{r}) + (n/y)^2 = 0$$

or, better still,

$$pq^2r^{-1} + n^2y^{\ 2} = 0$$

but

$$\text{á} = \frac{\mathrm{B}(\mathrm{E_o}) + \mathrm{c}}{\mathrm{f_1} + \mathrm{f_2}}\mathrm{f}(\text{ù})$$

is easier to read than

$$\alpha = \{[\mathrm{B}(\mathrm{E_o}) + \mathrm{c}]/(\mathrm{f_1}+\mathrm{f_2})\}\mathrm{f}(\omega)$$

or

$$\alpha = \{[\mathrm{B}(\mathrm{E_o}) + \mathrm{c}](\mathrm{f_1}+\mathrm{f_2})^{-1}\}\mathrm{f}(\omega)$$

(2) Do not mix built-up and slashed forms unnecessarily within one equation. Write

$$\frac{\mathrm{a}}{\mathrm{b}} = \frac{\mathrm{B}(\mathrm{E_o}) + \mathrm{c}}{\mathrm{f_1} + \mathrm{f_2}}\mathrm{f}(\text{ù})$$

instead of

$$\mathrm{a/b} = \frac{\mathrm{B}(\mathrm{E_o}) + \mathrm{c}}{\mathrm{f_1} + \mathrm{f_2}}\mathrm{f}(\text{ù})$$

On the other hand, a built-up fraction within the numerator or denominator of another built-up fraction is ungainly and hard to set. Write

$$\mathrm{B}' = \frac{[3\mathrm{J}/(\mathrm{J}+1)]\mathrm{T_N m^2}}{\mathrm{En} - \mathrm{Bn^2b^2n^2T_N}}$$

and, similarly,

$$\mathrm{A} = \frac{\mathrm{x}}{\mathrm{y}}\begin{pmatrix} (\mathrm{a}+\mathrm{b})/\mathrm{n} & 0 \\ 0 & (\mathrm{a}+\mathrm{b})/\mathrm{n} \end{pmatrix}$$

Use of a negative exponent is often the best alternative when slashing fractions within built-up fractions would be very awkward:

$$\mathrm{B}' = \frac{[3\mathrm{J}/(\mathrm{J}+1)]\mathrm{T_N m^2}}{\mathrm{En} - \mathrm{Bn^2} - \mathrm{b^2n^2T_N}}\left(\frac{\mathrm{B}(\mathrm{E_o}) + \mathrm{c}}{\mathrm{f_1} + \mathrm{f_2}}\mathrm{f}(\text{ù})\right)^{-1}$$

(3) As a rule, use built-up fractions in displayed equations containing integral, summation, or product signs. In displayed equations these signs are usually large, and therefore no particular economy results from using the solidus. Write

$$\frac{a}{b} = \int_0^{\partial} dx \quad \text{and} \quad \frac{a}{b} = \sum_{i=1}^{n} a_i$$

instead of

$$a/b = \int_0^{\partial} dx \quad \text{and} \quad a/b = \sum_{i=1}^{n} a_i$$

(4) Always use slashed fractions in subscripts and superscripts:

$$\int_z^{\partial/2} \left(\frac{t - z}{\hat{o}^{-\acute{a}/(1-\acute{a})}} \right)^{3/2} dt$$

(5) Any rational fraction can be put on one line in the special upright form. This form is awkward in subscripts and superscripts [see point (4) above]; elsewhere, use the upright form instead of forms with a solidus as often as possible. Write $\frac{1}{3}$ x instead of x/3, (1/3)x, or l/3 x; never write l/3x unless you mean l/(3x). Write

$$\sin\left(\frac{1}{3}x\right) \quad \text{and} \quad \frac{1}{3}\sin(x)$$

instead of

$$\sin(x/3) \quad \text{and} \quad (\sin x)/3$$

never write sinx/3.

(6) When using the solidus, make sure that your meaning is unambiguous:

a/b + c means (a/b) + c

but use the parentheses, or write ab-1 + c, to allay any doubts. Never write

a/b/c;

write instead

(a/b)/c or a/(b/c)

(7) Make sure that the fraction line clearly delimits the numerator and denominator of a built-up fraction. Does

$$\begin{matrix} \ln & a \\ & - \\ & b \end{matrix} \quad \text{mean} \quad \frac{\ln a}{b} \quad \text{or} \quad \ln\left(\frac{a}{b}\right)?$$

Do not leave the resolution of such ambiguities to copyeditors.

4. Integral, summation, and product signs

In running text, integral, summation, and product signs cannot be full sized, and symbols attached to them must always be subscripts and superscripts: $\int_C$, $\sum_{i=1}$, $\prod_a$ In displayed equations, integral, summation, and product signs can be full sized, and there is ample room for symbols above or below summation and product signs:

$$\int_C , \sum_{i=1} , \prod_a$$

D. RECOMMENDED NOTATION

1. Parentheses

(1) According to accepted convention, one works outward with parentheses according to the scheme

{[( )]}

Boldface parentheses are available; once the above scheme is exhausted, repeat the sequence in boldface. Also use boldface parentheses to enclose the argument of a function if the argument itself contains parentheses.

(2) Small parentheses and brackets are available for use in subscripts and superscripts. Oversize parentheses and brackets are available for use in displayed equations around expressions containing built-up fractions or integral, summation, or product signs.

(3) In general, too many parentheses are better than too few. But wholesale use of parentheses obscures rather than clarifies, and--especially in displayed equations--wastes labor and space. All of the parentheses in the following example are extraneous:

$$\left[\left(\frac{(1+2)}{a^2}\right)+\left(\frac{2x^2}{(0.01)\partial/a}\right)\right]=0$$

(4) The introduction of special brackets that do not simply group, but also define, what they enclose does not affect the usual scheme:

$$\left[\left|h\left(\langle E\rangle^{1/2}+m\right)\right|^2-\ddot{A}S\right]^{-1}$$

But if any of the common parentheses or brackets are used with a special meaning, they should not then be used for grouping:

$$\frac{1}{2}\left\{i\left(L_1[L_2,L_3]+K_j\right)\right\}$$

In such cases, it is wise to identify the special brackets when they first appear to assure copyeditors that the unusual ordering is no slip.

(5) The following special uses of parentheses and brackets are recommended:

plane, or set of parallel planes (111)
point designated by coordinates (x,y,z)
vector written in components (Hx ,Hy, Hz)
function of a function F(f(x))
direction [111]
class (group) of symmetry-equivalent planes {111}
class (group) of symmetry-equivalent directions (111)

2. Mathematical functions set in roman type

(1) Standard abbreviations for functions set in roman type are given in Appendix D. Apart from these standard functians, the use of multiletter symbols for single quantities (such as "TKE" for total kinetic energy, or "BR" for branching ratio) is improper.

There is usually no need to mark the standard functions for roman type [see point (3) of Sec. IV B 2]; but take care, three-vectors when using such functions, to respect the following conventions.

(a) A function is closed up with its argument:
TrQ, $\tan(\eta^2/\mu)$

(b) The function of a product may be written without parentheses:
sin xa means sin (xa).
But put parentheses around the product whenever there is a chance of confusion:
sin (x2a3/2)
Note, however, that an argument is assumed to stop as soon as another functio appears:
sinx cosa means (sinx) cosa.

(c) sinx/a is ambiguous. Write (sinx)/a or sin (x/a), depending on the intended meaning.

(d) An argument stops at a plus or minus sign:
sinx + a means (sinx) + a

(e) A plus or minus sign should never directly follow a function. Write
sin[- (x + a)] rather than sin--(x + a)

(2) The decision to use exp or its italic alternative e depends on the nature of the argument. Use e if the argument is simple enough to make a readable superscript:
eBlT, e -x2

If the argument is long, or contains complicated indices, a built-up fraction, or an integral, summation, or product sign, use exp instead:

$$\exp\left[-\mathrm{h(t)} + (\text{ù} - \text{å})t\right]\exp\left(x_{ii'}^{2}a^{3/2}\right)$$

$$\exp\left(\frac{a + bx}{c + dy}\right), \exp\left(-\sum_{R,S} h_{RS}(0)\right)$$

Be consistent. Try not to use exp and e in proximity or for the same argument.

3. Radicals and bars over groups of symbols

The "roof " of a radical or a bar can be set over simple expressions, but may be awkward over complicated ones. An exponent 1/2 or special brackets may then be used instead: $(a^2 + b^2)^{1/2}$, $(A^n\,B)_{av}$. A bare, "unroofed" radical may also be used, but always with a liberal use of parentheses to avoid ambiguity. Clearly, as substitutes for roofed radicals, forms with an exponent 1/2 are usually less troublesome and more readable than forms with a bare radical.

4. Multiplication signs

(1) Use a multiplication sign for the vector product of three-vectors:

$\overline{V}$ x A

As a rule, do not use multiplication signs for simple products. Write

2*ab* instead of 2 x *ab* or 2*a* x *b*

Exception is made for the continuation of a product from line to line [see point (1) of Sec. IV C 1], and for the following special cases:

3.4 x 10-6 eV, 3x2x 1 mm

1x3x5x ... x (2*n*--1), $2f \times \frac{1}{4} g$

(2) Centered dots may be used in compound units: 4 g.cm^{2}.s^{-2}. Their principal use is in inner products of vectors, dyadics, and the like:

k•p, l•g•s.

Do not use dots for simple products, or for any other kind of multiplication. Write

2*ab* instead of 2•ab or 2a•b.

5. Mathematical terms

The American Institute of Physics strongly recommends the usage of the following symbols:

approximately equal to	$\approx$
proportional to	$\propto$
tends to	$\rightarrow$
asymptotically equal to; of the order of magnitude	$\sim$
of the order of (in the mathematical sense)	O ( )
complex conjugate of A	A^*
Transpose of matrix A	$A^{\sim}$, A^T
unit vector **k**/k	$\hat{\mathbf{k}}$

Customary usage in physics (in contrast to mathematics) is to denote complex conjugation by an asterisk rather than an overbar, and Hermitian conjugation by a superscript dagger (not a plus sign!) rather than an asterisk. Transposition should be denoted by a superscript tilde, or, better, a roman T, not an overtilde.